

# Active Distribution Network Voltage Profile Optimization Using Mixed Integer Linear Programming

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**Abstract**— The quality of electrical energy delivered to the customers greatly depends on the voltage deviation. To avoid negative effects on equipment belonging to the distribution system operator or final customer, legal regulations and recommendations in different countries are enforced to ensure that the voltage level does not deviate from the prescribed tolerances. The operation of different voltage control devices is coordinated in such a way that the voltage delivered to medium/low voltage customers remains within the required limits, despite changes in voltage drop due to load variations, distributed generation (DG) production variations, and changes in network configuration. This paper presents a mixed-integer linear programming method for voltage profile optimization in active distribution networks through optimization of on/off load tap changers and DG reactive power support. The effectiveness of the proposed method in relation to other Volt-Var methods is tested on a benchmark network. The numerical results show advantages of the proposed approach which are manifested through voltage profile normalization and deviation reduction.

**Keywords**—voltage profile optimization, on-load tap changers, reactive power support, mixed integer linear programming

## NOMENCLATURE

### Sets

$T$  set of time instances / extreme scenarios  
 $N$  set of network buses  
 $N^{SP}$  set of supply points  
 $TR$  set of transformers  
 $PL$  set of power lines  
 $DG$  set of distributed generators

### Parameters

$g_{ij}$  conductance of line  $ij$   
 $b_{ij}$  susceptance of line  $ij$   
 $b_{ijo}$  shunt admittance of line  $ij$   
 $V_i^{min}|V_i^{max}$  minimum/maximum voltage limit for bus  $i$   
 $S_{ij}^{max}$  power rating of line  $ij$   
 $n$  number of segments used in the approximation  
 $t_{ij}^{min}$  minimum relative transformer ratio  
 $\Delta t_{ij}$  relative transformer ratio change per step

$K_{ij}$  number of transformer tap positions

### Variables

$\Delta V_i^{max}$  maximum absolute deviation of voltage magnitude at bus  $i$   
 $V_{i,t}$  voltage magnitude of bus  $i$  at time(scenario)  $t$   
 $V_{i,t}$  voltage magnitude of fictitious bus  $i$  at time(scenario)  $t$   
 $\Delta V_{i,t}$  voltage magnitude deviation of bus  $i$  at time(scenario)  $t$   
 $\Delta V_{i,t}^{pos}$  positive voltage magnitude deviation value of bus  $i$  at the time(scenario)  $t$   
 $\Delta V_{i,t}^{neg}$  negative voltage magnitude deviation value of bus  $i$  at time(scenario)  $t$   
 $\theta_{i,t}$  voltage phase angle of bus  $i$  at time(scenario)  $t$   
 $\theta_{ij,t}$  voltage phase angle difference of buses  $i$  and  $j$  at time(scenario)  $t$   
 $P_{ij,t}|Q_{ij,t}$  active/reactive power flow of line connecting buses  $i$  and  $j$  at time(scenario)  $t$   
 $P_{k,t}|Q_{k,t}$  active/reactive power flow of line  $k$  at time(scenario)  $t$   
 $t_{ij}$  relative OLTC tap ratio  
 $T_{ij}$  OLTC tap position  
 $\lambda_{ij,n}$  binary value used to represent OLTC tap position connecting buses  $i$  and  $j$

## I. INTRODUCTION

Although Distribution System Operator (DSO) has a large collection of methods and approaches for compensating the voltage drop across the distribution network, utilization of such equipment increases the complexity of operation and maintenance of the network. In EU at middle voltage levels e.g. 20 or 30 kV, voltage drop problems are rare, and control made by on-load tap changers (OLTCs) on HV/MV transformers is usually adequate. The OLTC is a device with discrete set of states in which transformer tap ratio is mechanically altered while the transformer is energized and is used to control the voltage of the substation MV busbar. In

passive distribution networks, without any distributed generation connected, the voltage level is highest at the MV busbar of the HV/MV substation and drops towards the end of the feeder under all loading conditions. Voltage drop is largest under heavy consumer load. In passive MV grids, where the selected MV level is lower (10 kV), voltage drop problems can be solved by installing booster transformers with OLTC along the feeder.

Since most consumers are connected to the low voltage grid, off-load taps on MV/LV transformers are used to flatten out the voltage in the LV network considering the MV/LV transformer position along the feeder. In case of off-load tap changers, tap ratio cannot be changed when the transformer is energized. This means that every tap ratio change requires an interruption of the electricity supply. Therefore, their position is decided at the planning stage and kept constant throughout the year/season.

Solar photovoltaics (PV) and wind turbine generators (WTG) integrated with the MV grid act as distributed generations (DG). In active distribution networks, with connected DGs, voltage levels can rise towards the end of the feeder due to possible reversed energy flow under low consumer load and high energy generation from DGs. This makes system control more complex with an increasing number of DGs connected. Usually, the reactive power from DG connected to a certain network bus can support network voltages only at neighboring buses. Therefore, DGs reactive power (VAR), combined with tap changers, can be used to control voltages in MV grids. Due to the complexity of the problem, Volt-VAR optimization algorithm is designed to find the optimal solution and minimize voltage deviations.

Different methods of Volt-VAR control, based on either centralized or local techniques, are proposed in recent papers. Authors in [1] present a voltage control method that combines central and local control using PV inverters. The proposed control ensures that the voltage complies with the limits, resulting with lower losses in relation to the local control methods. This method also reduces DG active power curtailment in relation to local control methods. In [2] authors propose a method that restricts real power production to prevent network overvoltages through real-time adaptive setting of power caps for PV inverters, while fairly distributing the real power curtailments among all the PV units in the network. This method does not require global information and can be implemented either in a centralized supervisory control scheme or in a distributed way via consensus control.

Many different approaches have been used to find optimal Volt-VAR regulation while minimizing power curtailment in active distribution systems with DG. An algorithm based on the sensitivity matrix decomposition for the optimal voltage regulation in such networks is proposed in [3]. When the voltage at particular nodes exceeds normal operating limits, the nearest DGs can be located and instructed to control the voltage. The proposed voltage regulation approach is suitable for large heavily-meshed distribution networks.

A most common cause for transformer maintenance is wear on the tap changer mechanism, therefore, reducing the number of tap operations is optimal. In [4] authors investigate the influence of the synchronous machine-based distributed generation (DG) effect on the Volt-VAR control.

Results show that coordination among OLTC, substation switched capacitors and feeder-switched capacitors decrease the number of OLTC operations, losses, and voltage fluctuations in distribution systems, with and without DG present. In [5] authors propose a suboptimal approach based on sequential convex programming (SCP). The proposed approach provides a near-optimal solution with much lower computation complexity (runtime). The risk-assessment approach to the reactive power planning problem is presented in [6]. Chance-constrained programming is used to model the random equivalent availability of existing reactive power sources for a given confidence level. In [7] a method to optimally set the tap position of voltage regulation transformers in distribution systems is proposed. Problem is cast as a rank-constrained semidefinite program (SDP), in which the transformer tap ratios are captured by introducing a secondary-side “virtual” bus per transformer, and constraining the values that these virtual bus voltages can take according to the limits on the tap positions. The tap positions are determined as the ratio between the primary-side bus voltage and the secondary-side virtual bus voltage obtained as the optimal solution of the relaxed SDP, which are then rounded to the nearest discrete tap values. To efficiently solve the relaxed SDP, distributed algorithm based on the alternating direction method of multipliers is proposed. In [8], a reactive power (VAR) optimization model that combines the precise linear modeling of OLTC is proposed to reduce power losses and voltage deviations. In the proposed method, the distributed generator, static and discrete VAR compensators, and transformer OLTC are chosen as control variables. The nonconvex optimization model is transformed into a convex problem through second-order cone relaxation and precise linear modeling of OLTC. The VAR optimization model is tested the 33-bus system using different scenarios with a various number of OLTC transformers. In [9] zone-based multistage “time-graded” operation of cascaded OLTC transformers, capacitor banks, and step voltage regulators in the presence of large-scale PV sources is introduced. A multistage Volt-VAR optimization algorithm is proposed to regulate the voltage in an MV unbalanced distribution system while trying to relax the tap/switch operations of regulators that are cascaded in series, and minimize the curtailment of PV inverter output. The method applied a zonal level-based management of regulating devices utilizing Volt/VAR control methods.

In this paper, we present a mixed-integer linear programming method for voltage profile optimization in active distribution networks through optimization of on/off load tap changers and DG reactive power support. The method is applied to a test case that considers a large set of different operating states. Given the size of the optimization problem, a method to identify the set of extreme operating scenarios is also illustrated. The rest of the paper is organized as follows. Section II. explains the mathematical formulation of the proposed approach. Section III. presents a case study and discusses the results. Finally, the relevant conclusions are provided in Section IV.

## II. VOLT-VAR OPTIMIZATION - MATHEMATICAL FORMULATION

### A. Linearized OLTC model

Fig. 1 shows a segment of a distribution network feeder with a generalized distribution network branch model connected between buses  $i$  and  $j$ . With adequate parameter selection, the generalized distribution network branch model can be used to model both power line ( $t_{ij}=1$ ) and OLTC ( $C_{ij}=0$ ).

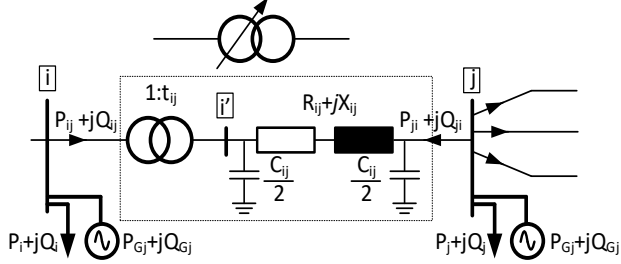


Fig. 1. Generalized distribution network branch model

In case when generalized distribution network branch model is used to model OLTC ( $C_{ij}=0$ ) it is necessary to introduce additional fictitious buses to account for the ideal transformer which can have a non-standard tap ratio. The voltage at fictitious bus  $i'$  can be expressed with:

$$V_{i'} = t_{ij}V_i \quad (1)$$

where  $t_{ij}$  represent relative OLTC tap ratio.

The equation (1) is non-convex but it can be transformed into mixed-integer linear expression using the approach described in [10].

$$t_{ij} = t_{ij}^{min} + T_{ij}\Delta t_{ij}, 0 \leq T_{ij} \leq K_{ij} \quad (2)$$

$$\Delta t_{ij} = (t_{ij}^{max} - t_{ij}^{min})/K_{ij} \quad (3)$$

Where  $t_{ij}^{min}$  and  $t_{ij}^{max}$  represent the minimum and maximum relative transformer ratio;  $K_{ij}$  represents the number of tap positions;  $\Delta t_{ij}$  represents relative transformer ratio change per step;  $T_{ij}$  represents the OLTC tap position. Based on the approach from [10], OLTC relative tap ratio  $t_{ij}$  can be expressed using binary codification as:

$$t_{ij} = t_{ij}^{min} + \Delta t_{ij} \sum_{n=0}^{N_{ij}} 2^n \lambda_{ij,n} \quad (4)$$

$$\sum_{n=0}^{N_{ij}} 2^n \lambda_{ij,n} \leq K_{ij} \quad (5)$$

$$V_{i'} = t_{ij}V_i = t_{ij}^{min}V_i + \Delta t_{ij} \sum_{n=0}^{N_{ij}} 2^n x_{ij,n} \quad (6)$$

$$x_{ij,n} = \lambda_{ij,n}V_i \quad (7)$$

Constraint (7) represents a nonlinear term due to a product of variables that can be linearized through equations (8) and (9) by introducing a positive large number  $M$ . Number  $M$  value is linked to the upper voltage limit at bus  $i$ :

$$0 \leq V_i - x_{ij,n} \leq (1 - \lambda_{ij,n}) \cdot M \quad (8)$$

$$0 \leq x_{ij,n} \leq \lambda_{ij,n}M \quad (9)$$

This linearized OLTC model is also used for modeling transformers with off-load tap ratio change whose static optimal tap position is also determined in the proposed optimization model.

### B. Voltage Profile Optimization - Mathematical Formulation

The objective function (10) aims at minimizing voltage deviation from the nominal value. This can be achieved through the minimization of a sum of squared voltage deviations across the whole network and time span:

$$\min \sum_{i \in N} \sum_{t \in T} \Delta V_{i,t}^2 \quad (10)$$

A similar objective can be achieved by minimizing the sum of maximum voltage deviations across all network busses:

$$\min \sum_{i \in N} \Delta V_i^{max} \quad (11)$$

where  $\Delta V_i^{max}$  is set with the following equations:

$$\Delta V_i^{max} \geq \Delta V_{i,t}^{pos} \quad \forall i \in N, t \in T \quad (12)$$

$$\Delta V_i^{max} \geq \Delta V_{i,t}^{neg} \quad \forall i \in N, t \in T \quad (13)$$

subject to:

$$V_{i,t} = 1 + \Delta V_{i,t} \quad \forall i \in N, t \in T \quad (14)$$

$$V_{i',t} = 1 + \Delta V_{i',t} \quad \forall i \in N, t \in T \quad (15)$$

$$\theta_{ij,t} = \theta_{i,t} - \theta_{j,t} \quad \forall i, j \in N, t \in T \quad (16)$$

$$\Delta V_{i,t} = \Delta V_{i,t}^{pos} - \Delta V_{i,t}^{neg} \quad \forall i \in N, t \in T \quad (17)$$

$$\Delta V_{i,t}^{pos} \geq 0 \quad \forall i \in N, t \in T \quad (18)$$

$$\Delta V_{i,t}^{neg} \geq 0 \quad \forall i \in N, t \in T \quad (19)$$

Active/reactive power flow equations for branch  $k$  connecting busses  $i$  and  $j$  are given with:

$$P_{ij,t} = P_{k,t} = V_{i,t}^2 g_k + V_{i,t} V_{j,t} (g_{ij} \cos \theta_{ij,t} + b_{ij} \sin \theta_{ij,t}) \quad (20)$$

$$Q_{ij,t} = Q_{k,t} = -V_{i,t}^2 (b_{ij} + b_{bij,0}) + V_{i,t} V_{j,t} (b_{ij} \cos \theta_{ij,t} - g_{ij} \sin \theta_{ij,t}) \quad (21)$$

By substituting (14) in equations (20) and (21) and neglecting higher-order terms we get:

$$P_{ij,t} = P_{k,t} \approx (1 + 2 \Delta V_{i,t}) g_{ij} + -(1 + \Delta V_{i,t} + \Delta V_{j,t}) (g_{ij} + b_{ij} \theta_{ij,t}) \quad (22)$$

$$Q_{ij,t} = Q_{k,t} \approx -(1 + 2 \Delta V_{i,t})(b_{ij} + b_{ij0}) \\ + (1 + \Delta V_{i,t} + \Delta V_{j,t})(b_{ij} - g_{ij}\theta_{ij,t}) \quad (23)$$

Equations (22) and (23) still contain nonlinear terms in form of a product of variables  $\Delta V_{i,t}$ ,  $\Delta V_{j,t}$  and  $\theta_{ij,t}$ . Given that these variables are relatively small, these higher-order terms can be neglected. Incorporating such approximations in a model, the active/reactive power flow for distribution network power lines can be expressed as:

$$P_{ij,t} = (\Delta V_{i,t} - \Delta V_{j,t})g_{ij} - b_{ij}\theta_{ij,t} \quad (24)$$

$$Q_{ij,t} = -(1 + 2 \Delta V_{i,t})b_{ij0} + \\ -(\Delta V_{i,t} - \Delta V_{j,t})b_{ij} - g_{ij}\theta_{ij,t} \quad (25)$$

$$P_{ji,t} = -(\Delta V_{i,t} - \Delta V_{j,t})g_{ij} + b_{ij}\theta_{ij} \quad (26)$$

$$Q_{ji,t} = -(1 + 2 \Delta V_{j,t})b_{ij0} + \\ +(\Delta V_{i,t} - \Delta V_{j,t})b_{ij} + g_{ij}\theta_{ij,t} \quad (27)$$

$$\forall (i, j) \in PL, t \in T$$

For branches that contain OLTC, the equations are slightly modified to account for a fictitious bus (Fig. 1):

$$P_{ij,t} = (\Delta V_{i,t} - \Delta V_{j,t})g_{ij} - b_{ij}\theta_{ij,t} \quad (28)$$

$$Q_{ij,t} = -(1 + 2 \Delta V_{i',t})b_{ij0} + \\ -(\Delta V_{i,t} - \Delta V_{j,t})b_{ij} - g_{ij}\theta_{ij,t} \quad (29)$$

$$P_{ji,t} = -(\Delta V_{i',t} - \Delta V_{j,t})g_{ij} + b_{ij}\theta_{ij,t} \quad (30)$$

$$Q_{ji,t} = -(1 + 2 \Delta V_{j,t})b_{ij0} + \\ +(\Delta V_{i',t} - \Delta V_{j,t})b_{ij} + g_{ij}\theta_{ij,t} \quad (31)$$

$$\forall (i, j) \in TR$$

Constraints (32) and (33) enforce the active and reactive power balance at each bus (except for fictional buses used in the OLTC branch model).

$$P_{i,t}^{DG} - P_{i,t}^L = \sum_{j \in B} P_{ij,t} \quad \forall i \in N, t \in T \quad (32)$$

$$Q_{i,t}^{DG} - Q_{i,t}^L = \sum_{j \in N} Q_{ij,t} \quad \forall i \in N, t \in T \quad (33)$$

Distributed generation also participates in distribution network voltage control. The reactive power limits inside which a specific DG unit can operate are given with (34) and (35) and they are proportional to the DG unit available active power.

$$-Q_{i,t}^{DG,max} \leq Q_{i,t}^{DG} \leq Q_{i,t}^{DG,max} \quad \forall DG, t \in T \quad (34)$$

$$Q_{i,t}^{DG,max} = \tan(\cos^{-1} \varphi_i^{DG}) \cdot P_{i,t}^{DG} \quad \forall DG, t \in T \quad (35)$$

Equations (36-39) are used to limit bus voltages within their permissible limits, while constraints (40-41) are used to fix bus voltage magnitude and angle in reference buses to specified values.

$$V_i^{min} \leq V_{i,t} \leq V_i^{max} \quad \forall i \in N, t \in T \quad (36)$$

$$(V_i^{min} - 1) \leq \Delta V_{i,t} \leq (V_i^{max} - 1) \quad \forall i \in N, t \in T \quad (37)$$

$$V_i^{min} \leq V_{i,t} \leq V_i^{max} \quad \forall i \in N, t \in T \quad (38)$$

$$(V_i^{min} - 1) \leq \Delta V_{i,t} \leq (V_i^{max} - 1) \\ \forall i \in (i, j) \in TR, t \in T \quad (39)$$

$$V_{i,t} = fix. \quad \forall i \in N^{SP}, t \in T \quad (40)$$

$$\theta_{i,t} = fix. \quad \forall i \in N^{SP}, t \in T \quad (41)$$

To assure normal operation of power lines and OLTC inside nominal power rating, we introduce additional constraints:

$$P_{ij,t}^2 + Q_{ij,t}^2 \leq (S_{ij}^{max})^2 \quad (42)$$

$$\forall (i, j) \in PL \cup TR, t \in T$$

Constraint (42) represented in a P-Q plane represents a circle with a radius  $S_{ij}^{max}$ . Equation (42) is non-linear but can be linearized by an n-sided convex regular polygon through the following expression:

$$\left( \sin\left(\frac{360^\circ l}{n}\right) - \sin\left(\frac{360^\circ}{n}(l-1)\right) \right) P_{ij,t} + \\ - \left( \cos\left(\frac{360^\circ l}{n}\right) - \cos\left(\frac{360^\circ}{n}(l-1)\right) \right) Q_{ij,t} + \\ - S_{ij}^{max} \cdot \sin\left(\frac{360^\circ}{n}\right) \leq 0 \quad (43)$$

where  $l=1,2,\dots,n$ . The larger number of polygon sides  $n$  gives a better approximation of constraint (42) at the expense of a higher computational burden.

### III. TEST CASE

The proposed optimal voltage regulation model is tested on a modified IEEE 33-bus system [11] that has been extended through the addition of transformers and DG units. The OLTC is connected between busses 1 and 2 while the off-load tap changer transformers were added between each load bus (2-33) and newly formed LV buses (34-65) shifting the load connection to LV transformer side. The OLTC has 31 tap positions ranging from 0.85-1.15 (step 1% nominal turns) while off load tap changers have a narrower range with only 5 tap positions (step 2.5% nominal turns) ranging from 0.95-1.05. The PV plants are all connected to MV distribution network to busses 2, 9, 17 and 29. The modified IEEE 33-bus system is shown in Fig. 2. The model presented in section II.b. was implemented in Python and solved using Gurobi solver.

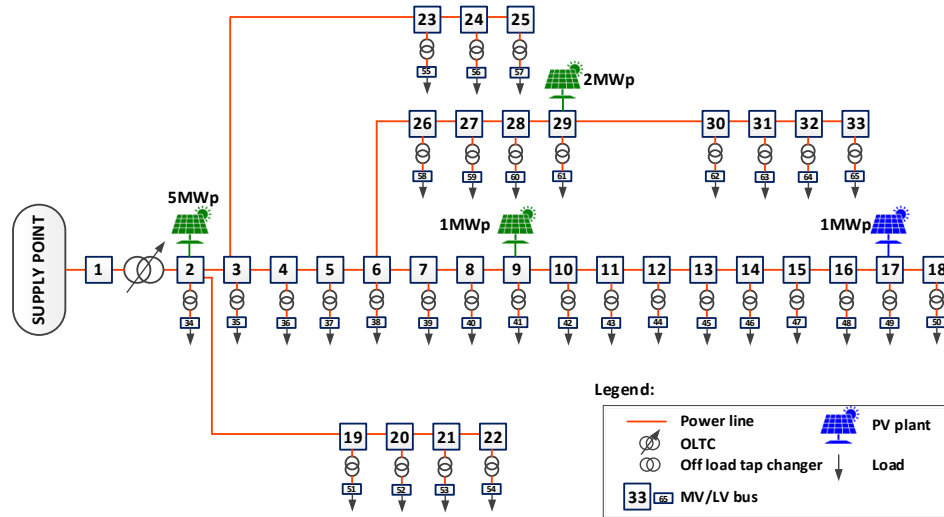


Fig. 2. Modified/extended IEEE 33-bus network

In order to optimize the distribution network voltage profile while considering different network operating conditions, in the analysis, we consider hourly load and DG production time-series data. The Fig. 3 shows the annual heatmap of network load and PV production as well as monthly consumption and DG production in relation to total

annual energy consumption / PV production. The load data represents a pattern typical for coastal tourist areas in Croatia with high consumption during summer time and low energy consumption during the remaining of the year. The PV production data is taken from the PV plants operating in the same area.

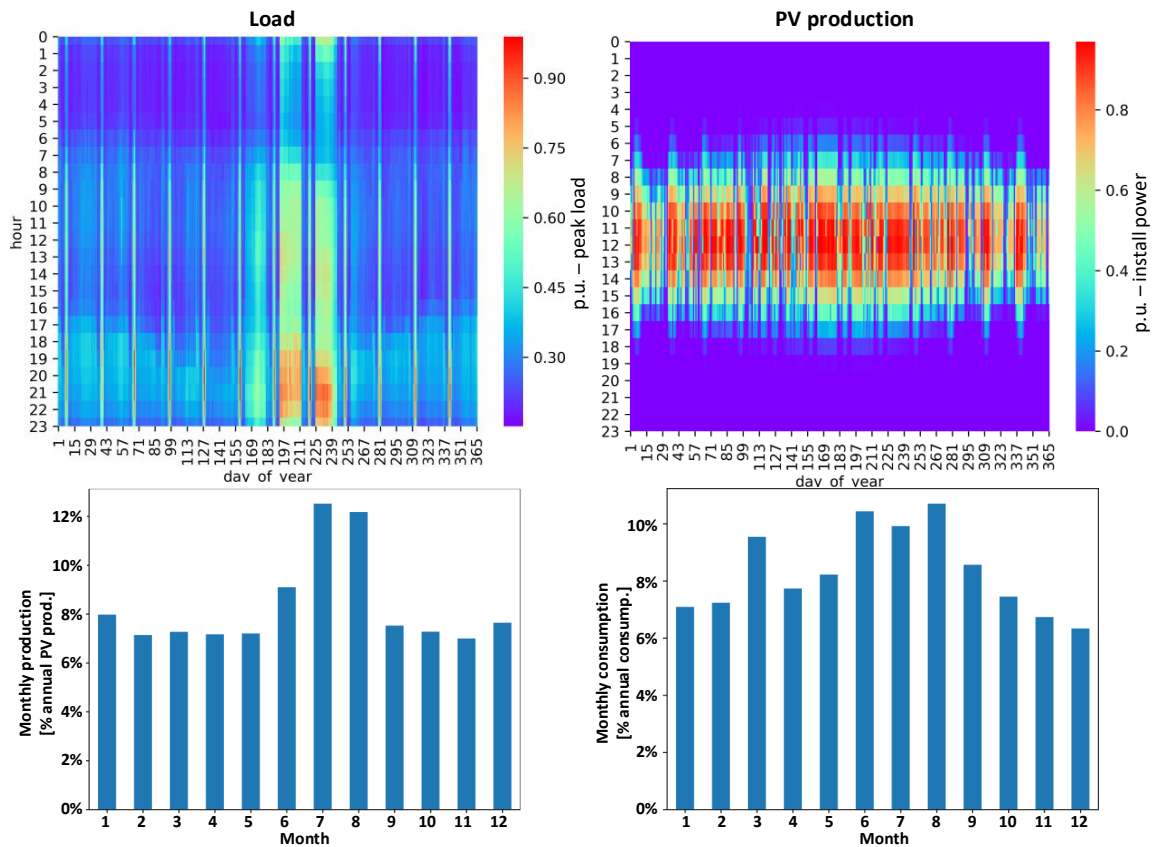


Fig. 3. Network load and PV production data

Running optimization problem defined in section II.b. which that time granularity and period duration would significantly increase model computational time. In order to

reduce the computational burden, in the proposed method we employ the scenario reduction technique which only considers extreme network operating conditions in terms of network load and DG production. Identification of an extreme set of network operating conditions that can lead to high/low voltage conditions in the distribution network is conducted by constructing a convex hull around network load/PV production data points in n-dimensional space. The vertices of the convex hull represent the set of extreme network conditions which are accounted for in the optimization problem. The basic illustration of the proposed approach is shown in Fig. 4 where the original data set of 8760 hourly data points is reduced to 62 extreme data points.

In order to identify the validity of the proposed approach as well as the influence of the PV production connection on the voltage conditions, several simulation scenarios were considered:

- **Case 1:** both OLTC as well as off-load tap changers are set to a neutral position (nominal turn ratio) while PV production units are operating with power factor  $\cos\phi=1$ ;
- **Case 2:** both OLTC as well as off-load tap changers are set in a neutral position (nominal turn ratio) while PV production units are operating with a power factor in a range  $0.95 \text{ cap.} < \cos\phi < 0.95 \text{ ind.}$  trying to maintain voltages at their point of connection equal to nominal values;
- **Case 3:** OLTC as well as off-load tap changer turn ratio is optimized together with PV unit power factor to minimize voltage deviations across the distribution network using the method described in section II.b..

8760 hourly data points -> 62 extreme points

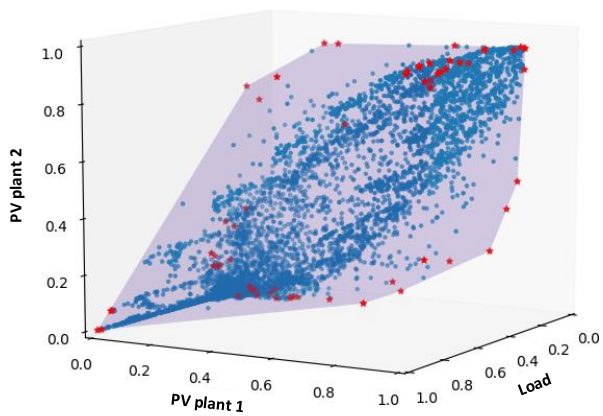


Fig. 4. Extreme load/PV production scenario selection

The summary of the results for three test cases considered in the analysis is given in the Table I. From the results, we can identify the main voltage problems in the network as well as the effect of different voltage control strategies on voltage profile improvement.

TABLE I. COMPARISON OF DIFFERENT VOLTAGE CONTROL METHODS ON NETWORK VOLTAGE CONDITIONS

	Mean voltage [p.u.]	Max voltage/Bus ID [p.u. / ID]	Min voltage/Bus ID [p.u. / ID]	Stand. dev. [p.u.]
<b>CASE 1</b>	0.9946	1.0975 / Bus_17	0.8877 / Bus_62	0.0231
<b>CASE 2</b>	0.9921	1.0661 / Bus_17	0.8877 / Bus_62	0.0182
<b>CASE 3</b>	1.0019	1.0610 / Bus_49	0.9558 / Bus_57	0.0138

The lowest voltage appears at bus 62 while the highest voltages appear at bus 17 where one of the PV plant is connected. In both Case 1 and Case 2 distribution network experiences low voltage problems in high demand periods (between 19:00-21:00h – see Fig. 3) when PV plant production is low or near zero value. Due to low or zero production of PV plant units in these specific critical periods, it is not possible to utilize reactive power support from PV plants to support the distribution network voltage profile. On the other hand, we can see that in Case 2, reactive power support from PV unit can add to voltage profile flattening which is evident through voltage standard deviation decrease in relation to Case 1.

In addition to PV plant reactive power support, additional voltage profile improvement can be achieved through optimal OLTC and off-load tap changer settings and operation. From Table I it is evident that by using the method described in section II.b. it is possible to improve low voltage conditions in critical operating points as well as to unify voltage profile even further. In Case 3, the standard deviation is additionally reduced and normal voltage conditions are assured even in critical peak hours (lowest voltage equal to 0.9558p.u.). The effect of voltage profile flattening for different test cases is shown on Fig. 5 and 6.

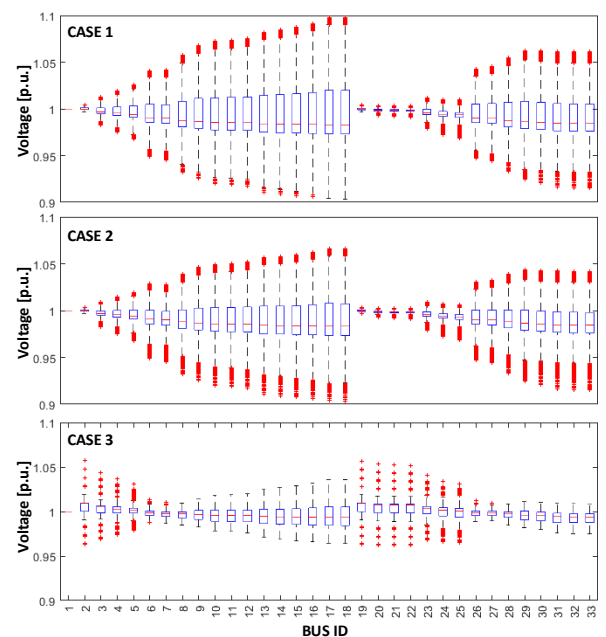


Fig. 5. Box plot of bus voltages - MV network buses

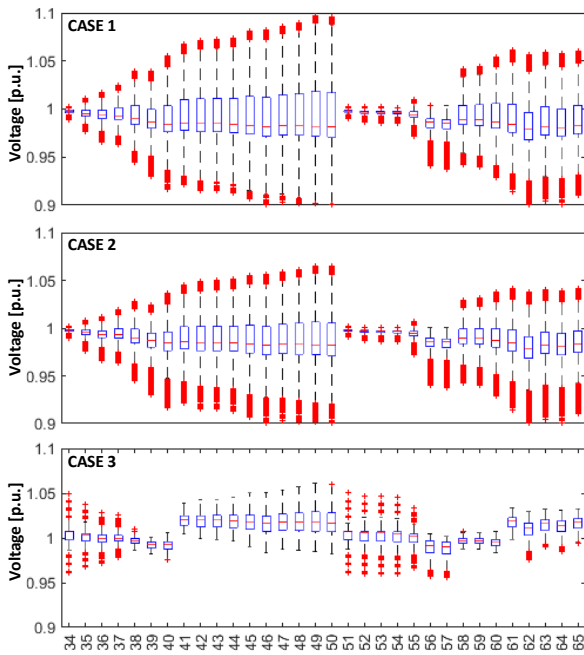


Fig. 6. Box plot of bus voltages - LV network buses

Although it is evident from Fig. 5. and Fig. 6. that combination of OLTC and DG reactive power support (Case 3) generally improves voltage profile, this control strategy can also result in larger voltage variations in certain parts of the network in relation to Case 1 and Case 2. From Fig.5. we can see that in buses near the main supply point and OLTC we have a wider band of bus voltages in Case3 than in Case 1 and Case 2. This is due to active OLTC control which helps to reduce voltage deviations in the other parts of the network. This is also present on sub-feeders 2-19-20-21-22 and 3-23-24-25 where we can also see a wider band of bus voltages in Case3 than in Case 1 and Case 2. Given that there are no DG plants on these sub-feeders, we don't have options such as DG reactive power control to compensate and reduce these voltage deviations. Despite the increase in voltage deviations in some parts of the network, the proposed approach generally improves voltage profile across the network achieving more uniform voltage conditions and reducing situations of extremely high/low voltage conditions.

#### IV. CONCLUSION

In this paper, we present a mixed-integer linear programming method for voltage profile optimization in active distribution networks through optimization of on/off load tap changers and DG reactive power support. The proposed method was tested on a modified benchmark IEEE network with one OLTC and 32 off-load tap changers as well as 5 PV production units. The proposed approach was compared with other different voltage control strategies frequently applied in active distribution networks. Test results show the advantages of the proposed approach against other methods in terms of significant voltage profile improvement and voltage deviation reductions.

The future work will be focused on integrating additional measures which can be used to improve distribution network voltage profile such as: topological reconfiguration, capacitor control, and load control. In addition to this, future work will analyze new challenges facing DSO related to electric vehicle integration and its effect on voltage conditions and network overload.

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