

**University of Split**  
**Faculty of Electrical Engineering, Mechanical**  
**Engineering and Naval Architecture**

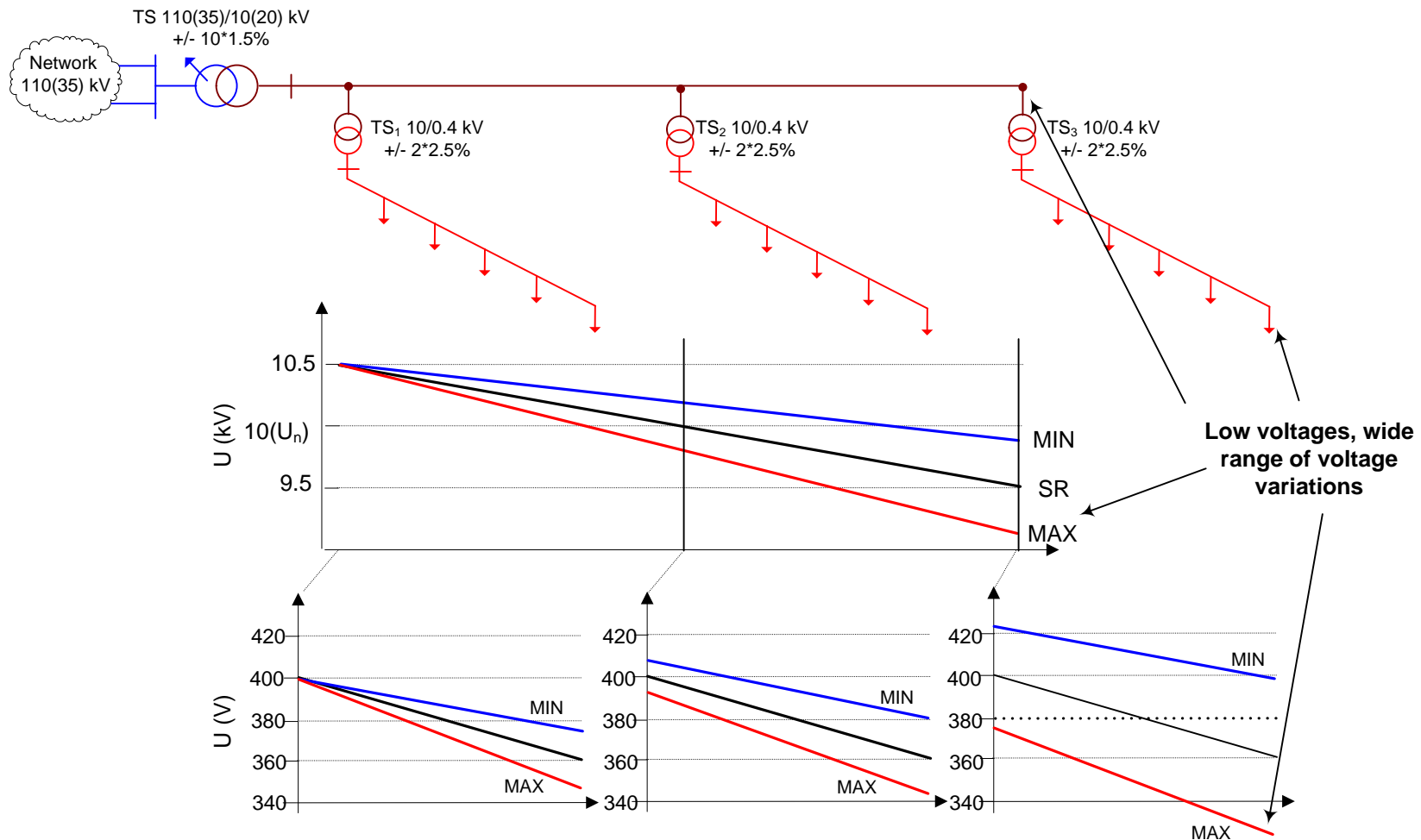


**Active Distribution Network Voltage Profile**  
**Optimization Using Mixed Integer Linear**  
**Programming**

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# The basic characteristics of passive distribution networks:

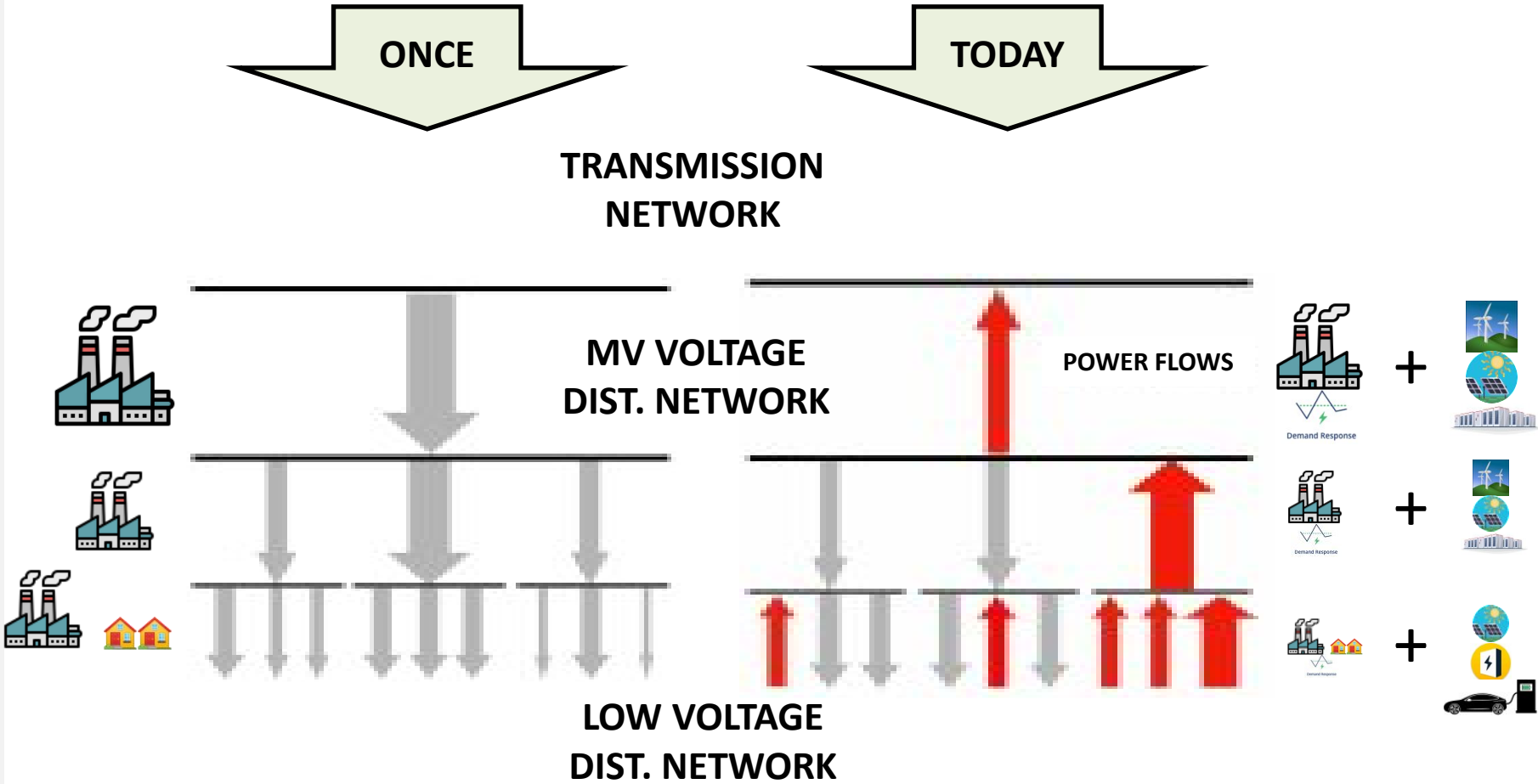


Limited possibility of voltage regulation (in the passive distribution networks):

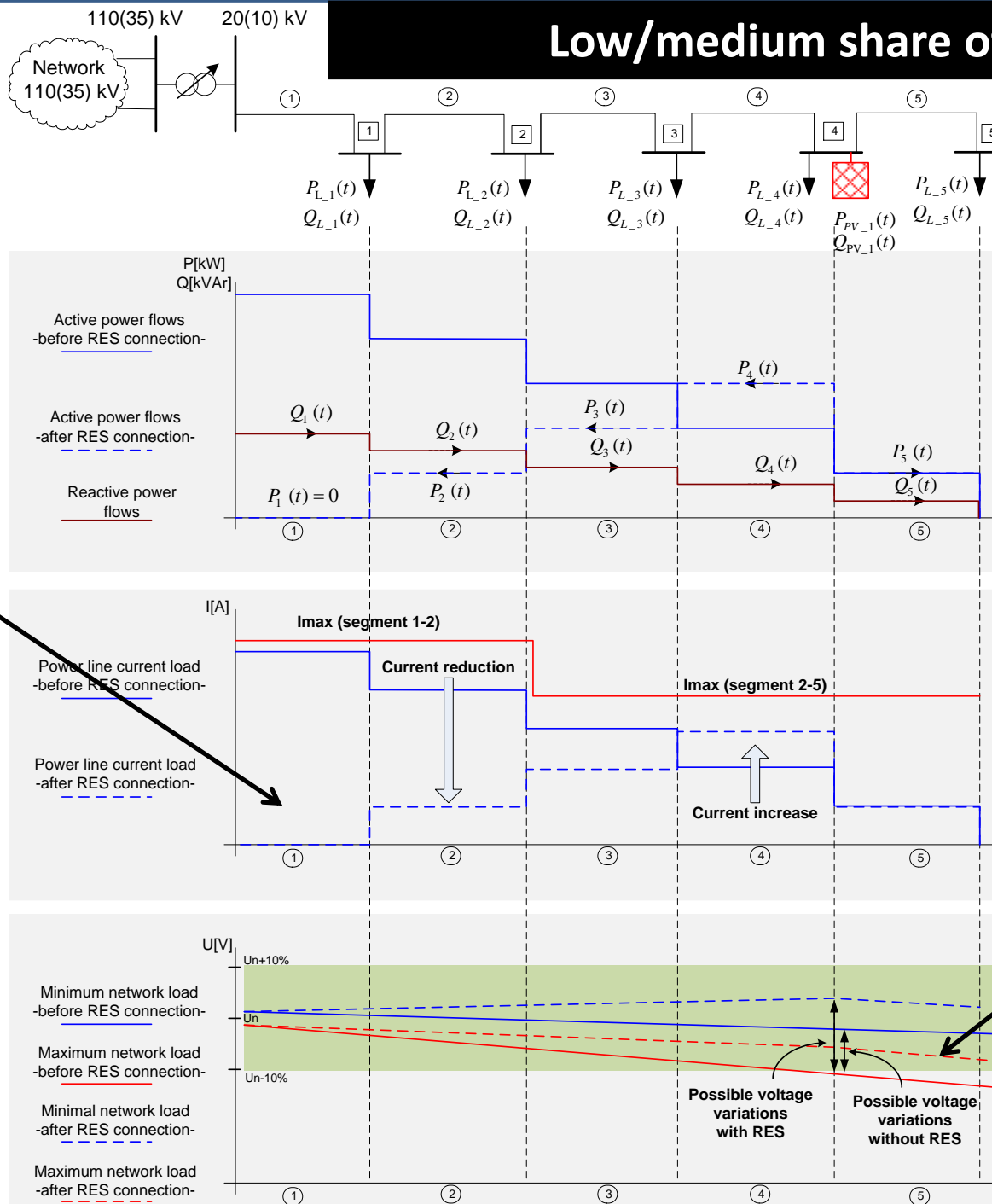
- transformers with on-load tap changers (110 /20(10 )kV);
- transformers with off-load tap changers (35/10kV, 20 (10) /0.4 kV);
- capacitor banks

# Active/passive distribution network

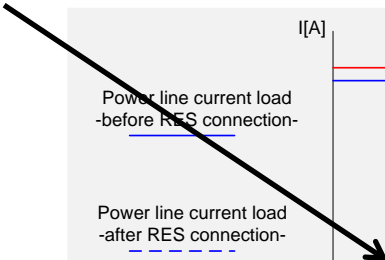
## Unidirectional/bidirectional power flows



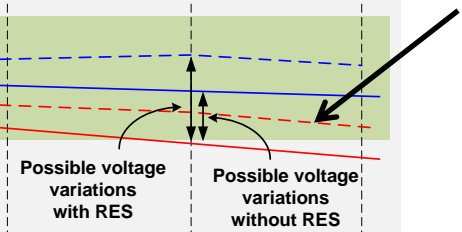
# Low/medium share of RES



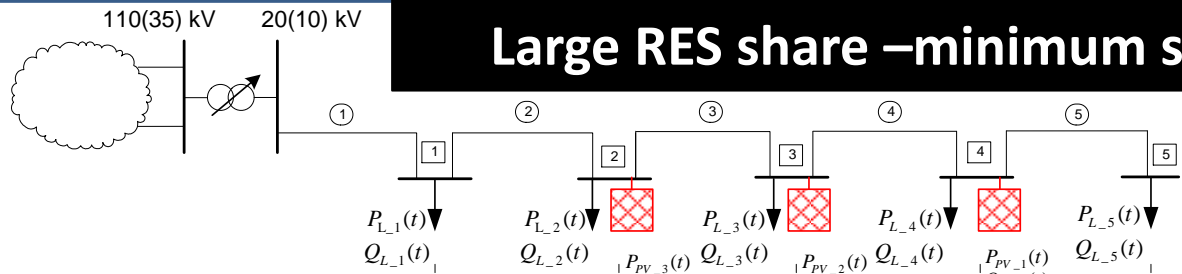
Increased network capacity - when the RES work



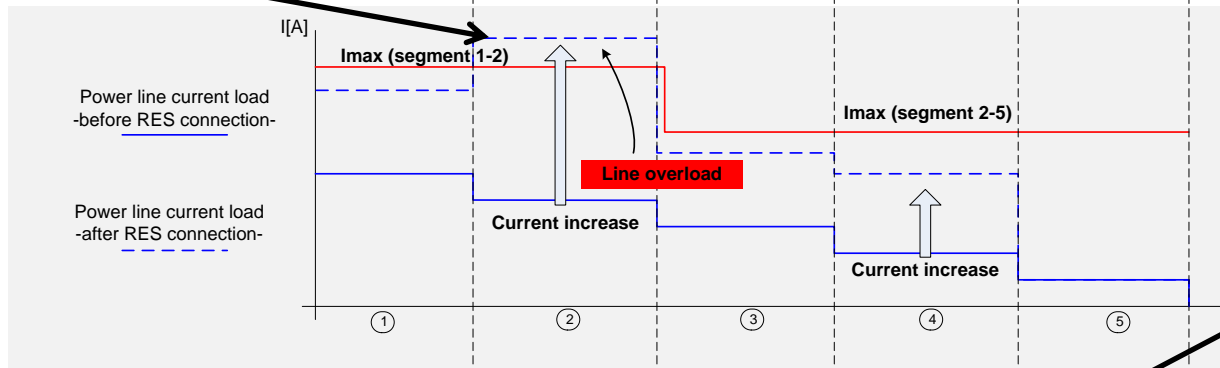
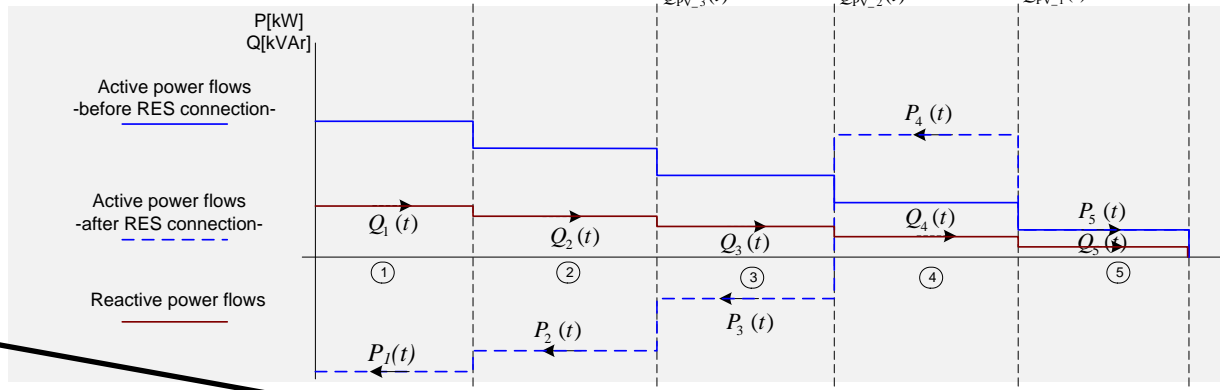
Better voltage conditions - when RES work ???



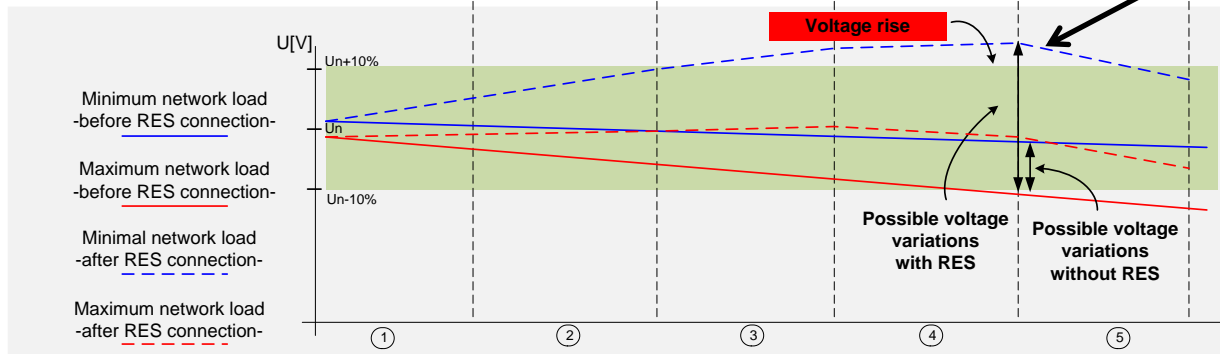
# Large RES share – minimum system load



**Line overloading**



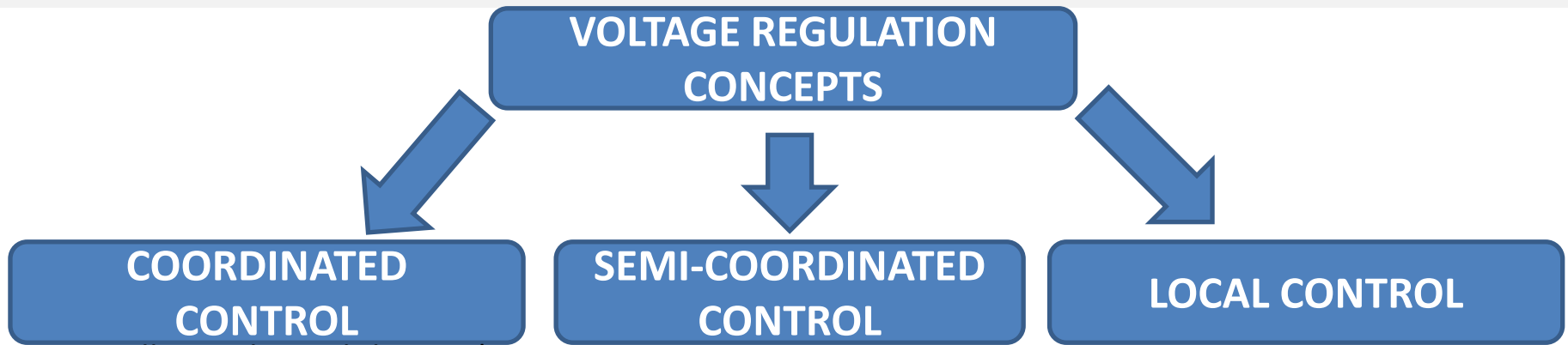
**Overvoltage**



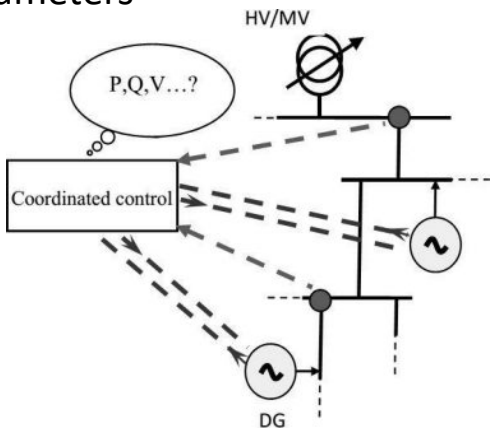
# Voltage regulation mechanisms:

- **Passive distribution network (=> problem with low voltages):**
  - Network reconfiguration
  - Network expansion and reinforcement
  - Transformers with variable number of turns:
    - on-load tap changers (110 / x kV)
    - off-load tap changers (35/10 kV, 10 (20) /0.4 kV, ...)
  - Capacitor banks (binary control, step control, continuous regulation)
- **Active distribution network (=> the problem of too high / too low voltages):**
  - Classic regulatory measures that are used in a passive distribution networks
  - Determination of the optimal network connection
  - The management of distributed electrical sources:
    - Active power regulation => active power curtailment as a measure to reduce network voltages
    - Reactive power control => production / consumption of reactive power as a measure to increase / decrease network voltage
  - Electricity storages
  - Electric Vehicles
  - Demand response,...

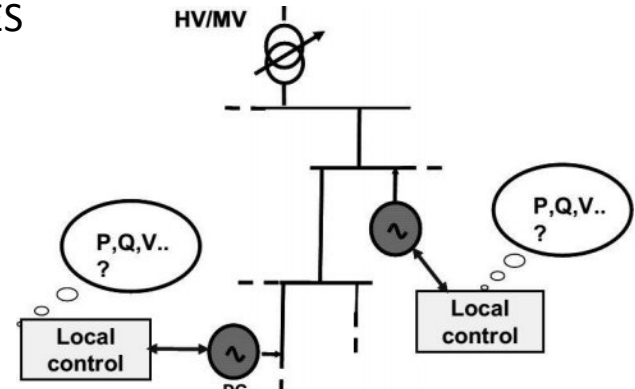
# Voltage regulation concepts:



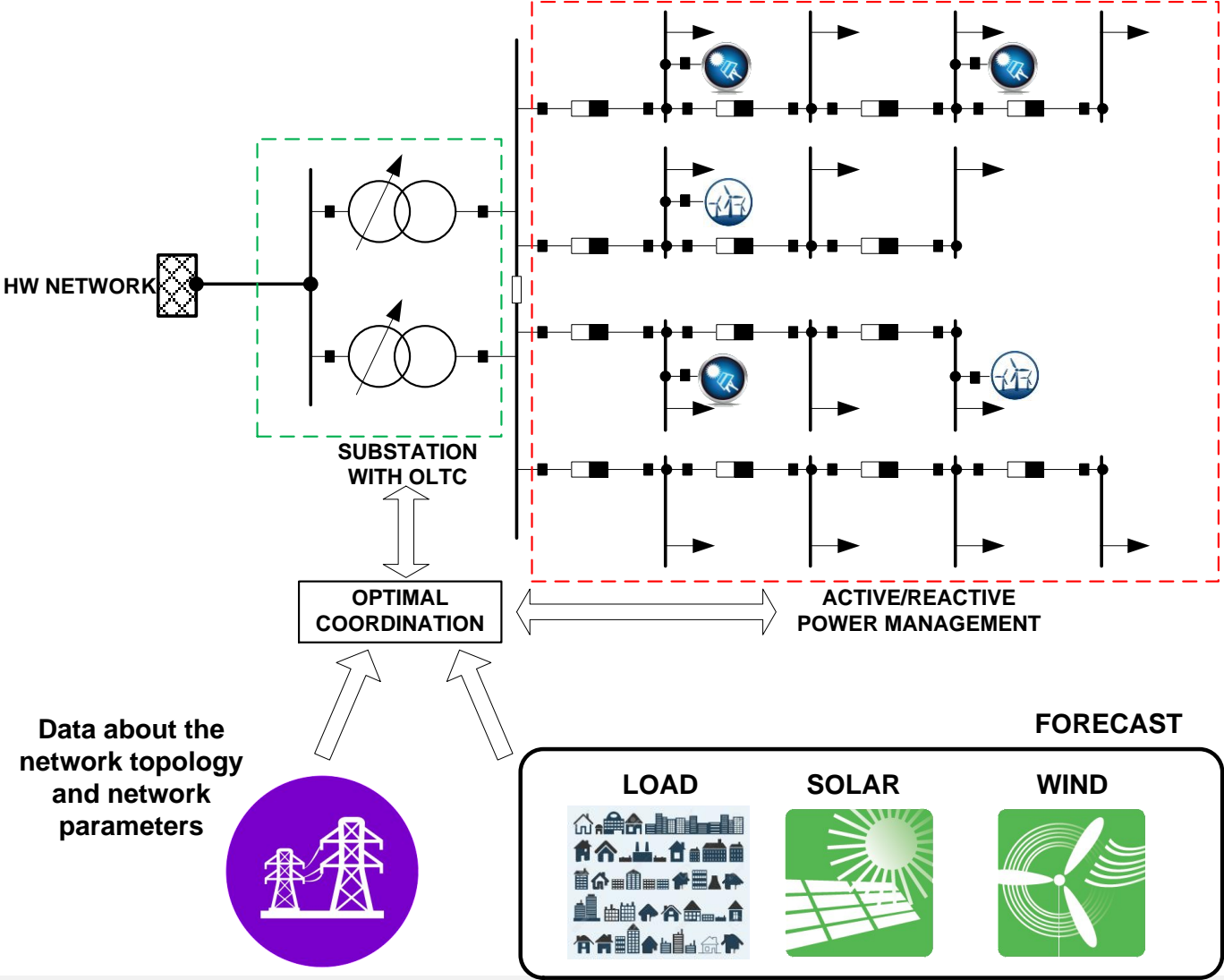
- Generally results with better (more optimal) network conditions
- Requires: highly developed IT infrastructure, good measurement coverage along distribution network, data defining network topology and its electrical parameters



- Generally gives poorer results
- No major requirements in terms of IT system;
- Interruptions in the communication system doesn't present threat for the system stability
- It requires only local measurements
- Automatic management at the level of RES



# Centralized control of the distribution networks with a high share of renewable energy





# Mathematical formulation– MILP

$$\min \sum_{i \in N} \sum_{t \in T} \Delta V_{i,t}^2 \Rightarrow \min \sum_{i \in N} \Delta V_i^{\max} \quad \Leftarrow \text{Minimize sum of absolute maximum node deviations}$$

$$\Delta V_i^{\max} \geq \Delta V_{i,t}^{\text{pos}} \quad \forall i \in N, t \in T$$

$$\Delta V_i^{\max} \geq \Delta V_{i,t}^{\text{neg}} \quad \forall i \in N, t \in T$$

$$V_{i,t} = 1 + \Delta V_{i,t} \quad \forall i \in N, t \in T$$

$$V_{i,t} = 1 + \Delta V_{i,t} \quad \forall i \in N, t \in T$$

$$\theta_{ij,t} = \theta_{i,t} - \theta_{j,t} \quad \forall i, j \in N, t \in T$$

$$\Delta V_{i,t} = \Delta V_{i,t}^{\text{pos}} - \Delta V_{i,t}^{\text{neg}} \quad \forall i \in N, t \in T$$

$$\Delta V_{i,t}^{\text{pos}} \geq 0 \quad \forall i \in N, t \in T$$

$$\Delta V_{i,t}^{\text{neg}} \geq 0 \quad \forall i \in N, t \in T$$

$$V_i^{\min} \leq V_{i,t} \leq V_i^{\max} \quad \forall i \in N, t \in T$$

$$(V_i^{\min} - 1) \leq \Delta V_{i,t} \leq (V_i^{\max} - 1) \quad \forall i \in N, t \in T$$

**Voltage constraints**

$$P_{ij,t} = (\Delta V_{i,t} - \Delta V_{j,t})g_{ij} - b_{ij}\theta_{ij,t}$$

$$P_{ji,t} = -(\Delta V_{i,t} - \Delta V_{j,t})g_{ij} + b_{ij}\theta_{ij,t}$$

$$Q_{ij,t} = -(1 + 2 \Delta V_{i,t})b_{ij} - (\Delta V_{i,t} - \Delta V_{j,t})b_{ij} - g_{ij}\theta_{ij,t}$$

$$Q_{ji,t} = -(1 + 2 \Delta V_{j,t})b_{ij} + (\Delta V_{i,t} - \Delta V_{j,t})b_{ij} + g_{ij}\theta_{ij,t}$$

$$\left( \sin\left(\frac{360^\circ l}{n}\right) - \sin\left(\frac{360^\circ}{n}(l-1)\right) \right) P_{ij,t} +$$

$$- \left( \cos\left(\frac{360^\circ l}{n}\right) - \cos\left(\frac{360^\circ}{n}(l-1)\right) \right) Q_{ij,t} +$$

$$-S_{ij}^{\max} \cdot \sin\left(\frac{360^\circ}{n}\right) \leq 0 \quad \forall (i,j) \in PL, t \in T$$

**Active/reactive power flows**

$$P_{i,t}^{DG} - P_{i,t}^L = \sum_{j \in B} P_{ij,t} \quad \forall i \in N, t \in T$$

$$QP_{i,t}^{DG} - Q_{i,t}^L = \sum_{j \in N} Q_{ij,t} \quad \forall i \in N, t \in T$$

**Node power balance**

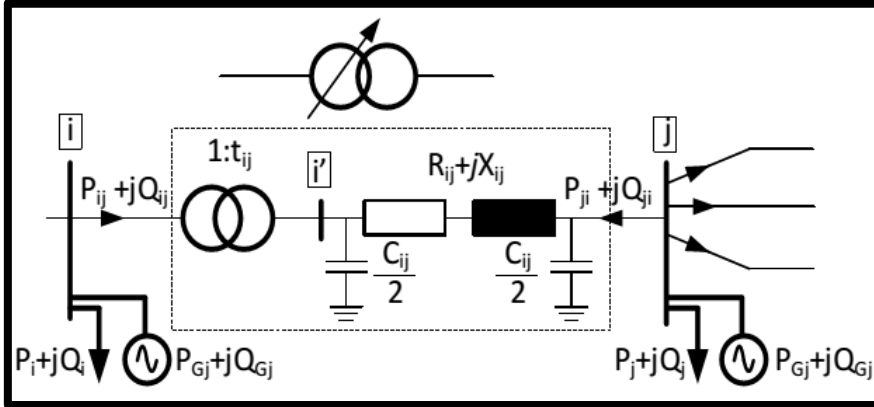
$$-Q_{i,t}^{DG,\max} \leq Q_{i,t}^{DG} \leq Q_{i,t}^{DG,\max} \quad \forall DG, t \in T$$

$$Q_{i,t}^{DG,\max} = \tan(\cos^{-1} \varphi_i^{DG}) \cdot P_{i,t}^{DG} \quad \forall DG, t \in T$$

**DG reactive power constraints**

# Mathematical formulation– MILP transformer modeling

Generalized distribution network branch model



$$V_{i'} = t_{ij}V_i$$

Linear transformer model with binary codification

$$t_{ij} = t_{ij}^{min} + T_{ij}\Delta t_{ij}, 0 \leq T_{ij} \leq K_{ij}$$

$$\Delta t_{ij} = (t_{ij}^{max} - t_{ij}^{min})/K_{ij}$$

$$t_{ij} = t_{ij}^{min} + \Delta t_{ij} \sum_{n=0}^{N_{ij}} 2^n \lambda_{ij,n}$$

$$\sum_{n=0}^{N_{ij}} 2^n \lambda_{ij,n} \leq K_{ij}$$

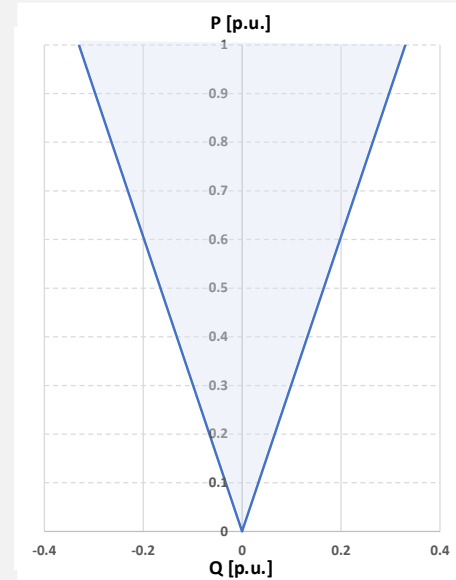
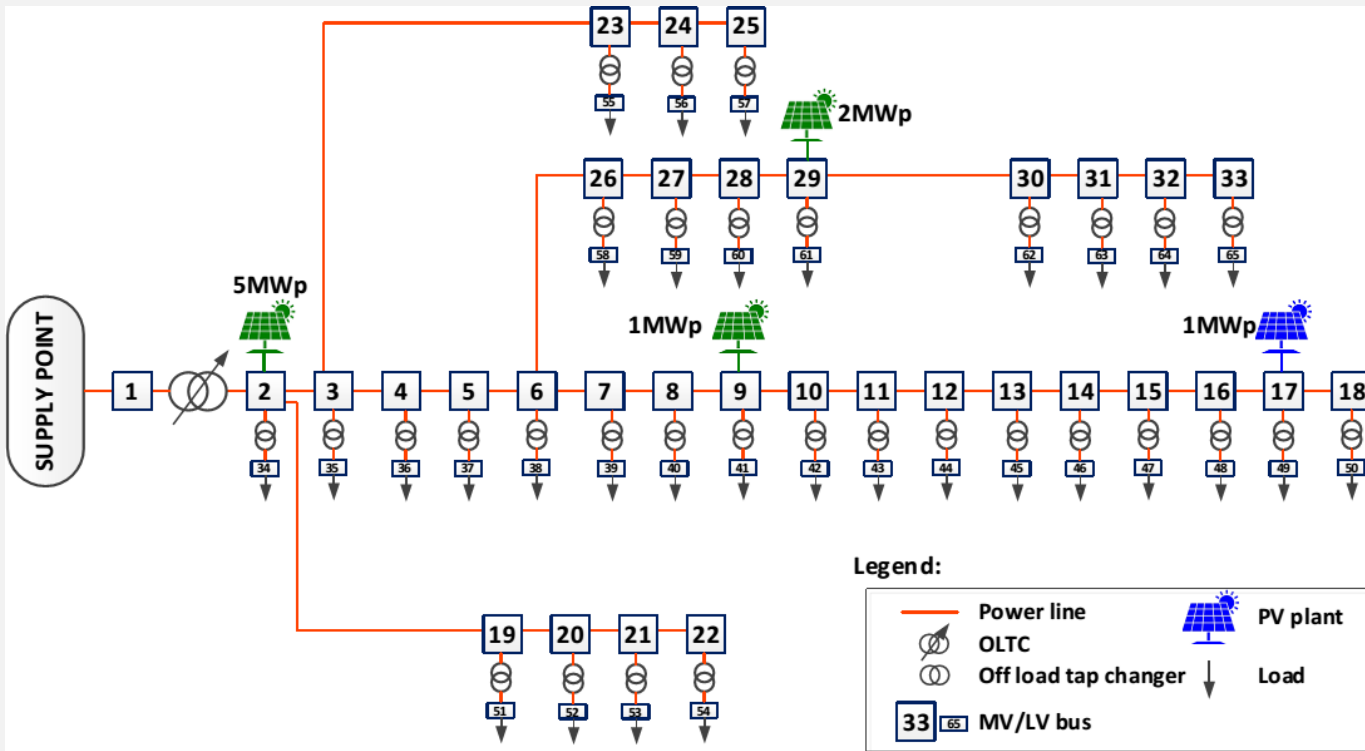
$$V_{i'} = t_{ij}V_i = t_{ij}^{min}V_i + \Delta t_{ij} \sum_{n=0}^{N_{ij}} 2^n x_{ij,n}$$

$$x_{ij,n} = \lambda_{ij,n}V_i$$

$$0 \leq V_i - x_{ij,n} \leq (1 - \lambda_{ij,n})M$$

$$0 \leq x_{ij,n} \leq \lambda_{ij,n}M$$

# TEST CASE – modified IEEE 33 bus model

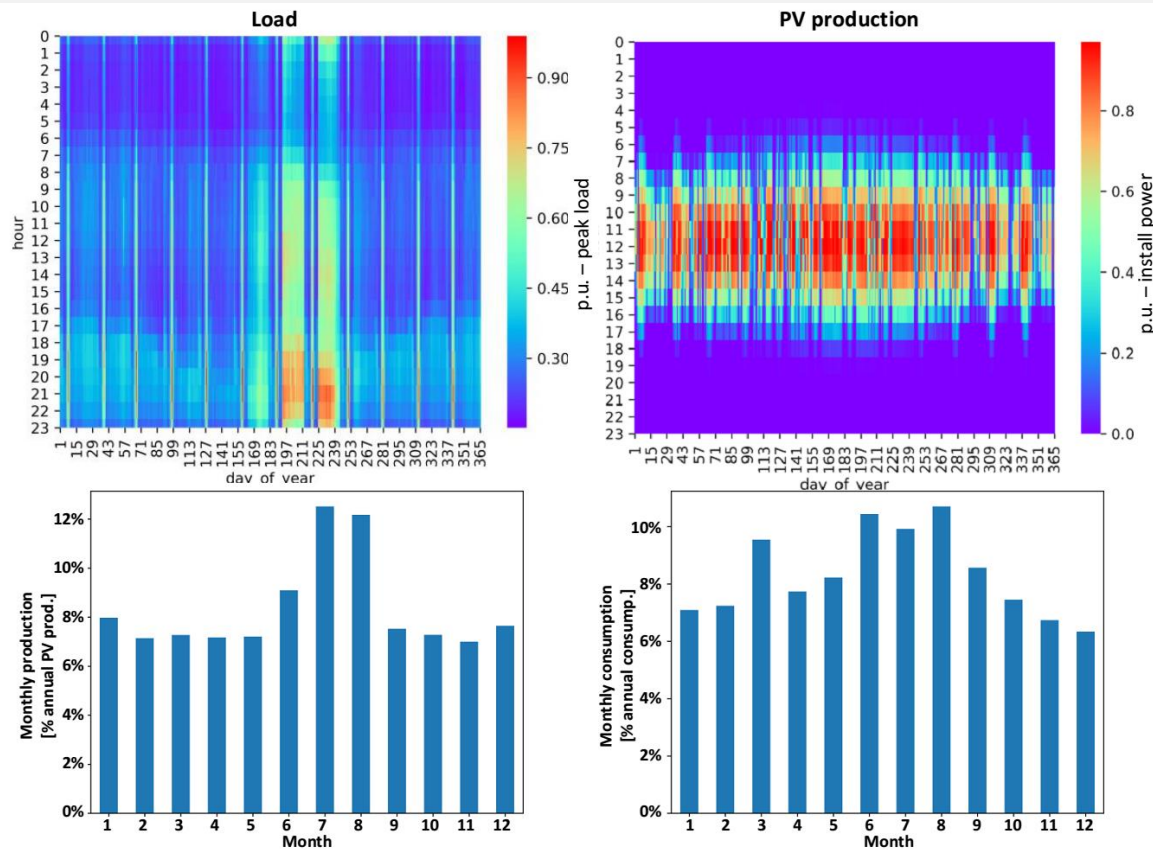


**DG reactive capability curve**

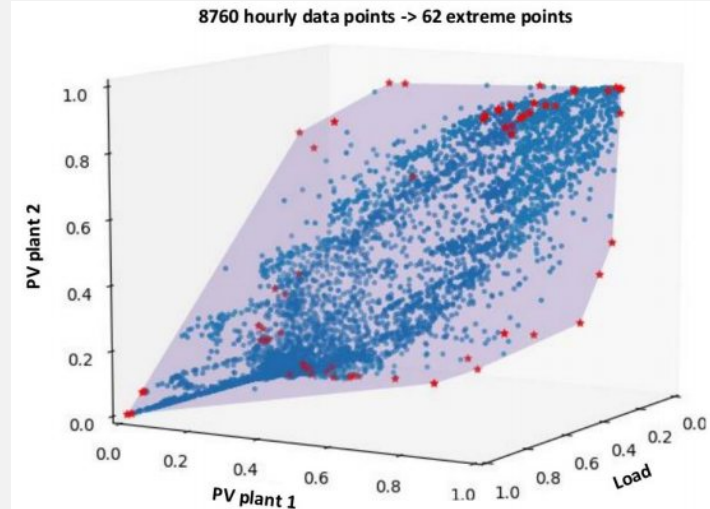
**Peak distribution network load:  
3.715 MW ; 2.3MVar**

**Total PV install power:  
9 MWp; max ( $\pm 2.95$ MVar)**

# Time series of consumption / RES production



Extreme load/PV production  
scenario selection

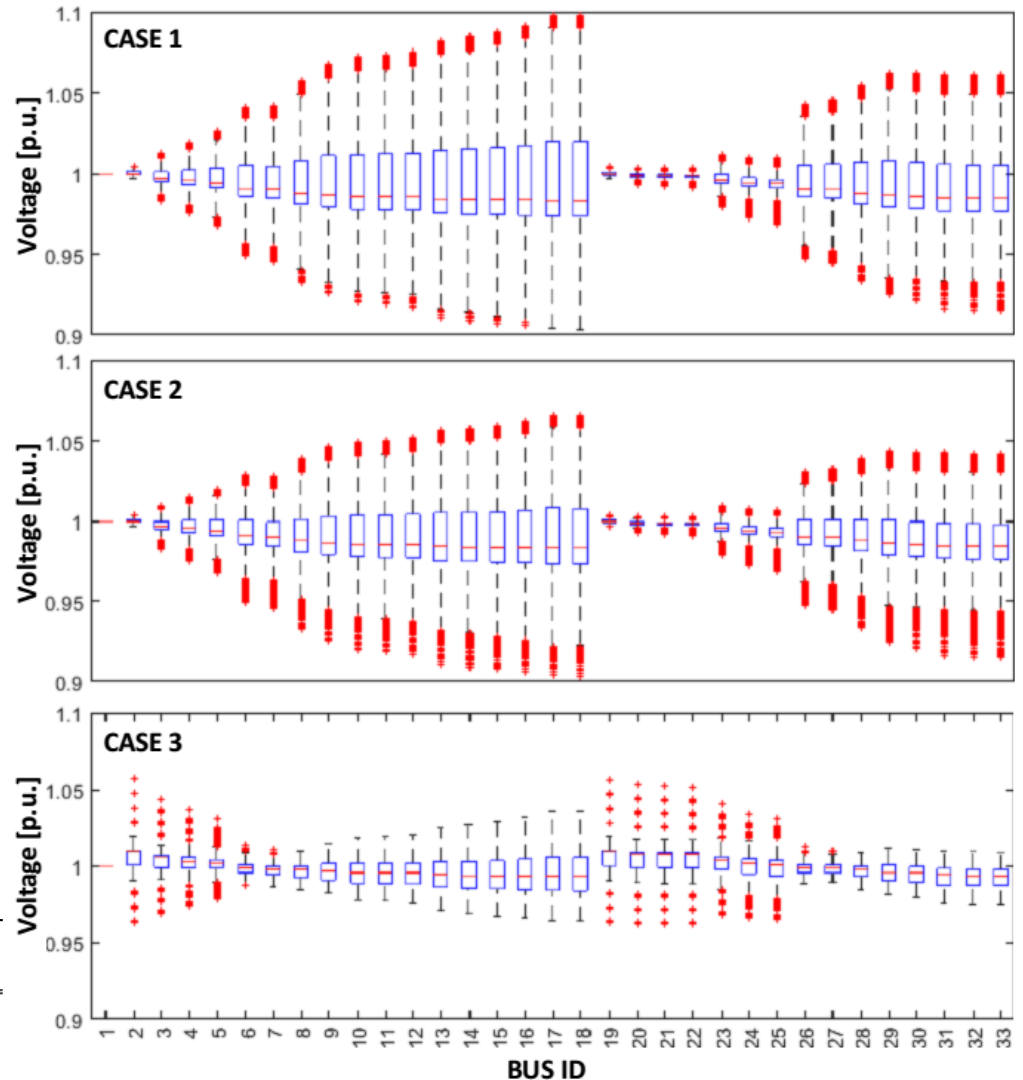


# TEST CASES & RESULTS

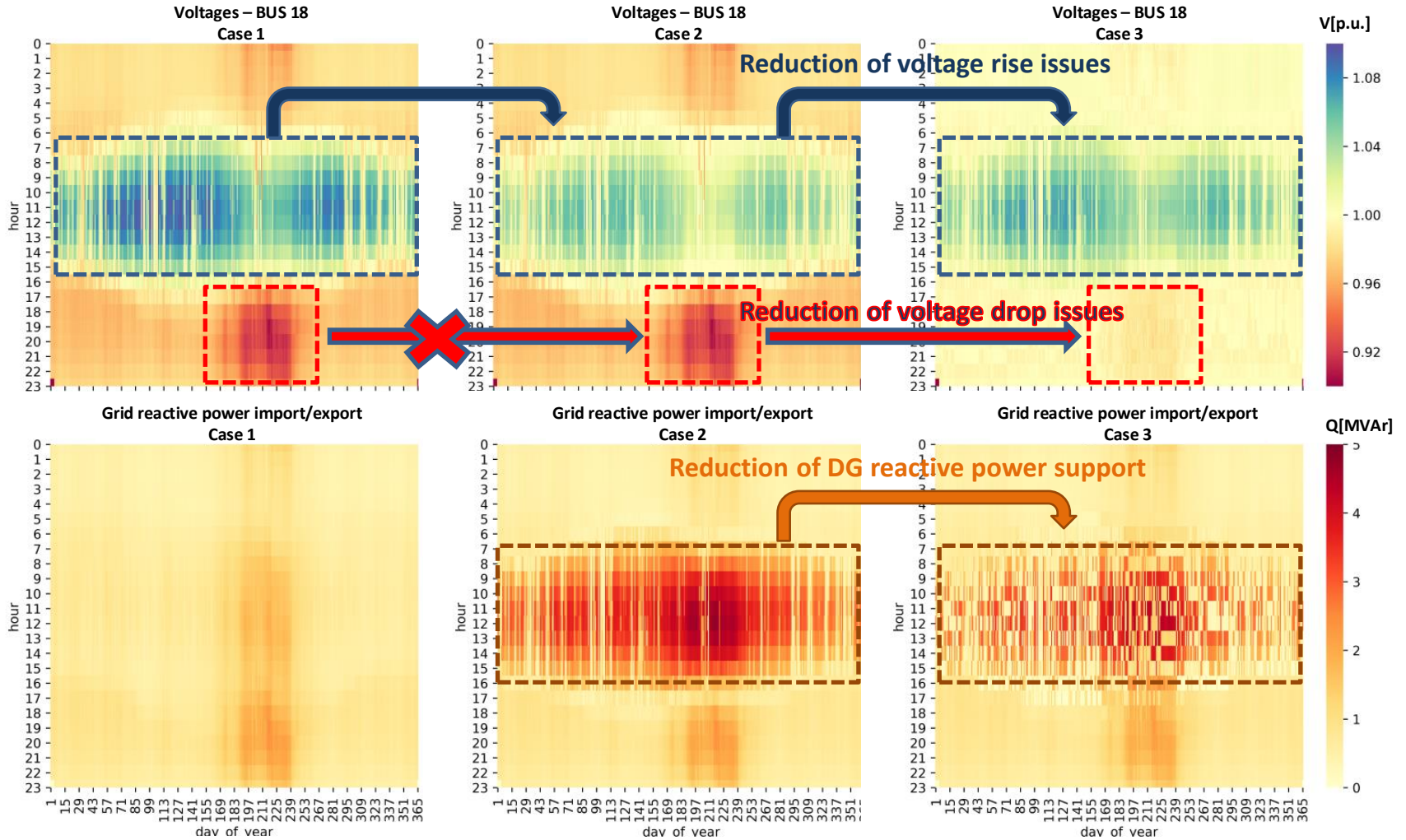
## Considered simulation scenarios:

- **Case 1:** both OLTC as well as off-load tap changers are set to a neutral position while PV plants are operating with power factor  $\cos\phi=1$ ;
- **Case 2:** both OLTC as well as off-load tap changers are set in a neutral position (nominal turn ratio) while PV production units are operating with a power factor in a range  $0.95 \text{ cap.} < \cos\phi < 0.95 \text{ ind.}$  trying to maintain voltages at their point of connection equal to nominal values;
- **Case 3:** OLTC as well as off-load tap changer turn ratio is optimized together with PV unit power factor to minimize voltage deviations across the distribution network using the method described in the paper

	Mean voltage [p.u.]	Max voltage/Bus ID [p.u. / ID]	Min voltage/Bus ID [p.u. / ID]	Stand. dev. [p.u.]
CASE 1	0.9946	1.0975 / Bus_17	0.8877 / Bus_62	0.0231
CASE 2	0.9921	1.0661 / Bus_17	0.8877 / Bus_62	0.0182
CASE 3	1.0019	1.0610 / Bus_49	0.9558 / Bus_57	0.0138

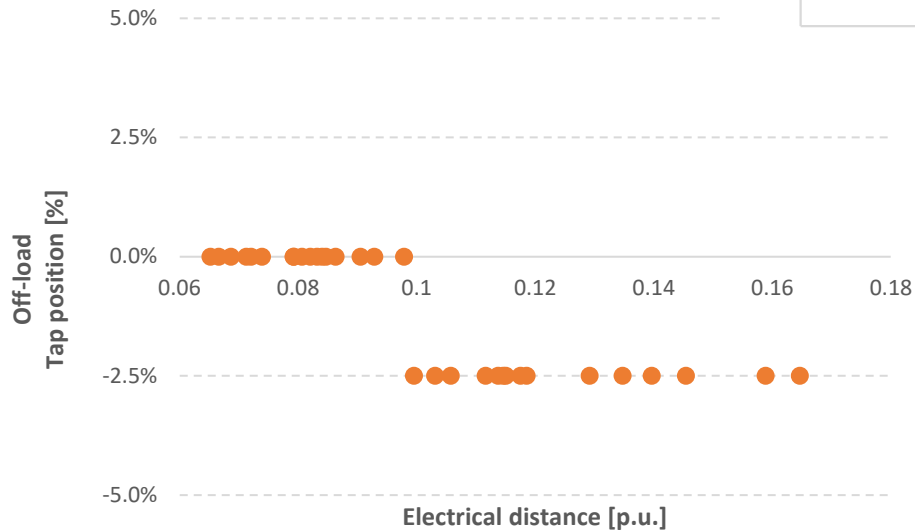
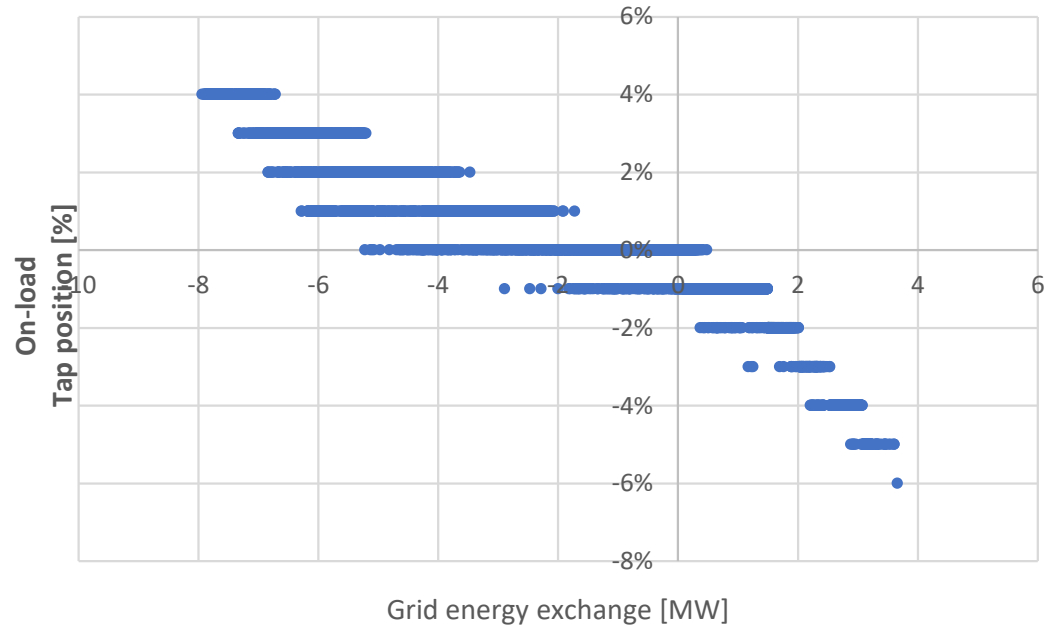


# TEST CASES & RESULTS



# TEST CASES & RESULTS – Case 3

**On-load tap changer operation  
in relation to grid energy exchange  
(Load – DG production)**



**Off-load tap changer settings  
in relation to bus electrical distances**



# Thank you for your attention!

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