Day-Ahead Scheduling and Real-Time Economic MPC of CHP Unit in Microgrid With Smart Buildings

Josip Vasilj, Sebastien Gros, Damir Jakus, and Mario Zanon

***Abstract*—This paper presents a model for day-ahead scheduling of the combined heat production heat and electric energy production for a residential microgrid taking into account the economic factors in a liberalized electricity markets, the technical factors in the safety/quality of supply, and the consumer preferences. This day-ahead scheduling model is complemented with a real-time economic model predictive control (MPC) model for a subsequent control with respect to the outcomes of the day-ahead scheduling. This combined scheduling and economic MPC provides a general set-up capable of overcoming several major difficulties encountered with a typical scheduling + tracking MPC set-up, e.g., the problems of connecting the economic objectives with different temporal resolution and different requirements in terms of delivery.**

***Index Terms*—Microgrid, day-ahead scheduling, economic MPC, smart buildings, stochastic programming.**

Nomenclature

The nomenclature used throughout the paper is defined hereafter.

Microgrid/Building Parameters

$R^{\mathrm{hs}}, C^{\mathrm{hs}}$	Thermal resistance and capacitance of the heat storage.
$R_b^{\mathrm{b}}, C_b^{\mathrm{b}}$	Building thermal resistance and capacitance, building b.
$A_b^{\mathrm{we}}, A_b^{\mathrm{w}}$	Effective and actual window area per building facade, building b.
k_p	Energy conversion coefficient for different processes, process p.
C^{fuel}	Cost of the fuel used in CHP process.

Optimization Variables

$Q_{t\xi}^{\mathcal{M}}; q^{\mathcal{M}}$	Heat associated with process $\mathcal{M}$ (*in*- turbine input heat, *ext*- heat extracted from the turbine, *h*- building heating system, *w*- water heating), period t, scenario ξ; Real-time heat output for the process $\mathcal{M}$.
$P_{t\xi}; p$	CHP electric power production, period t, scenario ξ; Real-time power output.
$P_{t\xi}^B; p^B$	Battery electric power production/consumption, period t, scenario ξ; Real-time power output.
$E_{t\xi}; e$	Battery energy content, period t, scenario ξ; Real-time power output.
$T_{t\xi}^{\mathrm{hs}}; \tau^{hs}$	Temperature of the water in heat storage, period t, scenario ξ; Real-time temperature.
$T_{t\xi b}^{\mathrm{in}}, \tau^{in}$	Average temperature in the building, period t, scenario ξ, building b; Real-time temperature.
$P_t^{\mathrm{grid}\pm}$	Power exchange between Microgrid and the main grid, period t.

Input Parameters

λ_t, k^{pr}	Day-ahead electricity market price forecast and buy/sell price ratio, period t.
$P_{t\xi}^{\mathrm{mg,rl}}$	Microgrid residual load forecast, period t, scenario ξ.
$T_{t\xi}^{\mathrm{out}}$	Outdoor temperature forecast, period t, scenario ξ.
$I_{t\xi}$	Solar irradiation forecast, period t, scenario ξ.

I. Introduction

THE LOCAL production and consumption of energy plays an essential role in the power system modernization. Several concepts became a standard among which a Microgrid is the most promising [1]. Although the Microgrid concept appears in various scales, an urban/rural settlement scale is particularly interesting for a Microgrid with combined heat production (CHP) unit. This scale allows for the exploitation of the economy of the scale benefits in heat supply compared to the household/building level. Therefore, a particularly interesting problem in this configuration is the combined management/control of the heat and the electric energy, i.e., taking into account their differences in technical and economical aspects. In this paper, we consider this issue and propose a model for co-optimization of electric and heating energy for day-ahead scheduling and subsequent real-time control of a CHP unit.

In order to determine the day-ahead trading quantities, it is necessary to forecast the parameters affecting the trading and to consider the possible deviations in these forecasts.

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For this purpose, a multi-stage stochastic optimization provides a suitable mathematical framework. In this scheduling, it is necessary to take into account the specificities of the CHP operation. Arroyo and Conejo [2] propose mathematical models for several technical aspects of the thermal unit operation participating on electricity spot market. A step further is taken in [3], where Houwing *et al.* propose a demand-responsive operation of a micro-CHP unit taking into account both electricity and heat requirements.

The challenge for real-time control is to avoid incurring in the cost of deviating from the day-ahead schedule while minimizing the cost of doing so. This control task depends on the actual realization of the stochastic variables, which is only approximated by the day-ahead prediction. Economic model predictive control (MPC) relies on optimal control in order to maximize the profit and is, therefore, a natural choice for CHP real-time control. First attempts at explicitly accounting for economics in real-time control can be found in [4]–[6]. In [4] a simple model is used to demonstrate the potential benefits of economic MPC. Halvgaard *et al.* [5] introduce an economic MPC scheme for climate control with respect to electricity costs. In [6], a similar problem is considered, but the focus is put on the distributed fashion in which the problem is solved. In both models, it is difficult to capture longer-term trading and periodic trajectories related to storages. The traditional way of handling this issue was a tracking MPC which would simply follow the trajectories determined with the scheduling, without directly optimizing for profit. Stability of economic MPC has been studied in [7] and [8], while the superior performance of economic MPC over tracking MPC has been demonstrated in [9] in a setting in which the optimal operating point was independent of the stochasticity of the problem. In our context, an additional difficulty arises, since tracking MPC would be based on scheduling performed with uncertain parameters and cannot easily include any information on the real-time costs. The setup we propose, instead, formulates the economic MPC in coordination with the day-ahead scheduling in order to capture phenomena occurring on the two different temporal scales. Therefore, both the trading, which is performed on a daily resolution, and the high-resolution real-time control can account for profit optimality.

The setup we propose is depicted in Fig. 1. In order to capture the possible trajectories of different states, the scheduling is also model-based. This is not the case in typical trading related scheduling. Furthermore, both the scheduling and the MPC are based on parameters forecasts, each on its temporal resolution.

This allows for the incorporation of a detailed building heat dynamics model that can be used for the simulation of real-time scenario realizations. This improvement is particularly valuable for the emerging concept of smart buildings[1] where it is necessary to incorporate the complex interactions between buildings and different supply systems accounting for

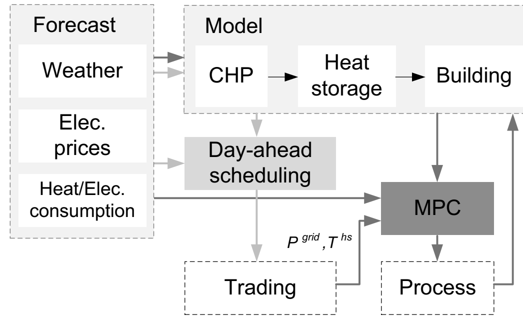

Fig. 1. Scheduling+MPC.

the weather and costumer requirements forecast (an example of smart building interaction with a smart grid can be found in [11]). The model presented here accounts for the requirements of the inhabitants and aims at maintaining the certain range of temperatures in buildings depending on the scenario realizations in forecasts.

On the electric side of the system, a Microgrid is assumed to have a certain renewable generation and the CHP unit scheduling has to take this into account. Based on these forecasts, together with the technical constraints resulting from the heat supply, the optimization algorithm determines the trading quantities. For this purpose, a liberalized electricity market is considered where electric energy trading is performed on the spot market, a day before the actual physical delivery. The Microgrid is modelled as a balancing responsible party (BRP) in a one-price imbalance settlement system. A thorough overview of the relations between typical market participants in a liberalized environment formulated through mathematical optimization can be found in [12].

Finally, a very important benefit of the set-up proposed here is the possibility of communicating the value of the stored heat in a heat storage from long-term scheduling to real-time control. Since the storages typically exhibit periodical patterns on the temporal scales for which they were designed, significantly lower temporal scales can not capture these longer phenomena. The combination of scheduling and real-time control resolves this issue.

This paper is organized in the following manner. The first section gives a brief description of the mathematical modelling of the stochastic input parameters. The second section describes the mathematical models used for the description of the energy conversion process from the CHP input heat to the building heat dissipation. In the third section, an optimization model for the day-ahead scheduling is given. Finally, the last section provides a model validation on a case study.

II. Physical Systems Modeling

This section gives an overview of the physical systems considered in the model. It is important to emphasize that the set-up proposed here is not limited to this system configuration but provides a general framework. However, we focus on this system since it captures an omnipresent configuration in urban settlements and there is a great potential for this kind of improvement.

[1]Smart building is a modern concept with the basic idea in passive-to-active transformation [10]. Technical basis for this transformation lays in the introduction of different sensors and actuators in the building management which, together with the communication infrastructure, enable the improvement in the management.

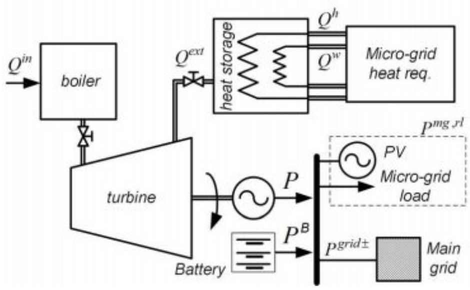

Fig. 2. CHP-based Microgrid.

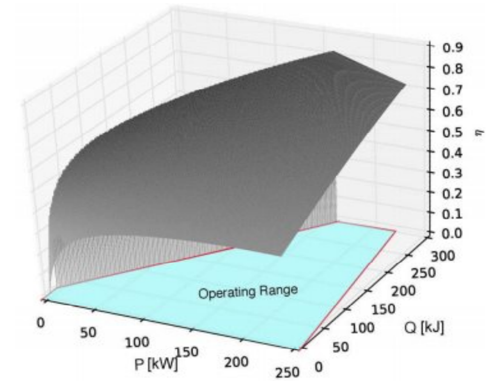

Fig. 3. Efficiency of the energy conversion.

There is a great variety of technical solutions for combined heat and power production, depending on the size, the fuel and the primary purpose. Fig. 2 depicts the CHP cycle used in this paper as a case study. The model presented here adopts the general characteristics describing all types of CHP systems and can be easily modified for different CHP system configurations.

A. Power and Heat Output

The power and heat output of a CHP unit is controlled through the position of the valves adjusting the flow of the steam from the boiler to the turbine and from the turbine to the heat extractor. While the dynamics of these processes can be quite complicated depending on the CHP unit design, we assume a simple CHP unit without re-heat and crossover, fully characterised by the produced power P and heat Q^{ext} extracted from the turbine, with following linear dynamics:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{T_{\text{CHP}}}\left(P^{\text{ctr}} - P\right), \tag{1a}$$

$$\frac{\mathrm{d}Q^{\text{ext}}}{\mathrm{d}t} = \frac{1}{T_{\text{CHQ}}}\left(Q^{\text{ctr}} - Q^{\text{ext}}\right), \tag{1b}$$

where P^{ctr} and Q^{ctr} are control signals which are followed by the CHP power P and heat Q^{ext} output adjustment according to the CHP electric and heat energy conversion process dynamics determined with their respective time constants T_{CHP} and T_{CHQ}.

The turbine heat input Q^{in} is eliminated from the system dynamics, since it can be computed using the turbine energy conversion equation which yields

$$Q^{\text{in}} = \frac{P + Q^{\text{ext}}}{\eta(P, Q^{\text{ext}})} \tag{2}$$

As shown in Figure 3, the efficiency η depends on P and Q^{ext} in a highly nonlinear way and the CHP operating range is bounded.

B. Heat Storage

The heat storage allows one to store heat during the times of cheaper/efficient production in order to use it later, during the expensive/inefficient production. An example of heat storage control and modeling can be found in [13]. This unit utilizes the heat energy extracted from the turbine (Q^{ext}) and provides the heat for residential and water heating. This system is described by the temperature T^{hs} of the medium used for the heat conservation, with dynamics

$$\frac{\mathrm{d}T^{\text{hs}}}{\mathrm{d}t} = \frac{1}{R^{\text{hs}}C^{\text{hs}}}\left(T^{\text{out}} - T^{\text{hs}}\right) + k_1 Q^{\text{ext}} - k_2\left(Q^{\text{h}} + Q^{\text{w}}\right), \tag{3}$$

where T^{out} is the outside air temperature, Q^{h} and Q^{w} denote respectively the heat extracted for building and water heating.

C. Building Model

The heat dynamics of building i are illustrated in Figure 4 and given by [14]:

$$\frac{\mathrm{d}T_b^{\text{in}}}{\mathrm{d}t} = \frac{1}{R_b^{\text{b}}C_b^{\text{b}}}\left(T^{\text{out}} - T_b^{\text{in}}\right) + \frac{1}{C_b^{\text{b}}}\left(k_3 Q_b^{\text{h}} + k_4 A_b^{\text{we}} I\right). \tag{4}$$

The heat absorbed through the windows is a product of a conversion factor k_4, the irradiance I, and the effective window area A^{we}, given by

$$A_b^{\text{we}}(t) = \sum_{m=0}^{3} A_b^{\text{w}} \frac{-\cos\left(\alpha^{\text{s}}(t) - \alpha^{\text{tilt}} + m \cdot 90°\right)}{\tan\gamma^{\text{s}}(t)}, \tag{5}$$

where α^{tilt} is the angle between the building orientation and the north, α^{s} and γ^{s} denote the azimuth and zenith angles and m is the side of the building. Details on the model identification and parameter estimation can be found in [14].

The total required heat can be obtained as a sum of the heat requirements of all buildings plus the heat network losses:

$$Q^{\text{h}} = \sum_b Q_b^{\text{h}} + Q^{\text{loss}}. \tag{6}$$

The heat network losses Q^{loss} are assumed to be constant since they are only a fraction of the total heat circulated in the system. However, it is worth mentioning that the steam flow through the pipes exhibits a highly non-linear behaviour which determines the losses.

D. Battery Model

Together with the heat storage an electricity storage is considered as well in the form of battery. Compared to the temporal scales of the heat conversion processes, battery dynamics

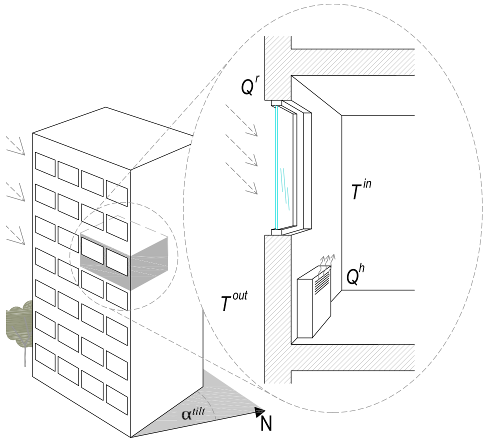

Fig. 4. Building parameters.

can be considered as instantaneous. Hence, the following equation, describing the energy content of a battery, provides a sufficient model of a battery:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = P^{\mathrm{B}}. \tag{7}$$

E. Energy Balance

The microgrid's electric power residual consumption $P^{\mathrm{mg,rl}}$, defined as the load minus the renewable generation, is covered by the local production of the CHP unit P and the battery storage P^B while the power shortfall/surplus is bought/sold on the market $P^{\mathrm{grid}\pm}$.

This balance can be summed up in the following equation:

$$P^{\mathrm{grid}+} - P^{\mathrm{grid}-} = P + P^B - P^{\mathrm{mg,rl}}. \tag{8}$$

F. The Aggregate Model

In the following, we lump the parameters in vector $\Theta = (T^{\mathrm{out}}, I, P^{\mathrm{mg,rl}}, T^{\mathrm{in,min}}, T^{\mathrm{in,max}}, Q^{\mathrm{w}}, A^{\mathrm{we}}, \lambda)$, the system's states in vector $X = (P, Q^{\mathrm{ext}}, T^{\mathrm{hs}}, T^{\mathrm{in}}, \Delta)$, the controls in vector $U = (P^{\mathrm{ctr}}, Q^{\mathrm{ctr}}, P^{\mathrm{grid}+}, P^{\mathrm{grid}-}, Q^{\mathrm{h}}, P^{\mathrm{B}})$, where Δ is an auxiliary variable whose meaning will be explained in the optimisation problem formulation. Then, we express the system dynamics as $\dot{X} = f(X, U, \Theta)$ and the system constraints as $h(X, U, \Theta) \leq 0$. Function h includes the operating range constraint depicted in Figure 3, the comfort constraint $T^{\mathrm{in,min}} \leq T^{\mathrm{in}}(t) \leq T^{\mathrm{in,max}}$, the heat storage constraint $T^{\mathrm{hs,min}} \leq T^{\mathrm{hs}}(t) \leq T^{\mathrm{hs,max}}$ and $P^{\mathrm{grid}+} \geq 0$, $P^{\mathrm{grid}-} \geq 0$.

III. Optimization Model

A CHP owner participating in the day-ahead market aims at gaining maximal profit, given the specificities of the market: all power-producing actors decide how much power to sell to the grid over a period of 24 hours, based on the market prices forecasts. Once the decision is taken, they have committed to deliver the chosen power profile and pay a cost for deviating from it. Therefore, we split the problem in two stages: the day-ahead scheduling and the real-time control.

In the first stage, a stochastic optimal control problem is solved in order to decide on the power profile to commit to, while accounting for the system dynamics and the parameter stochasticity. In the second stage, both (a) the economics of the problem have changed (since the CHP owner has committed to deliver a specific power profile) and (b) more up-to-date forecasts of the stochastic parameters are available. The problem in the second stage is therefore of a different nature from the one in the first stage. By viewing this as a decision-making problem, in the first stage a decision is taken only on P^{grid} taking into account the possible evolution of the system in the second stage. In the second stage (real-time) a decision is taken on all other variables.

A. Day-Ahead Scheduling

The CHP owner decides on the day-ahead market quantities by minimising the costs while maximising the profit, i.e., by minimising the functional

$$\begin{aligned} &J_{\mathrm{s}}(X(\cdot), U(\cdot), \Theta(\cdot)) \\ &:= -\int_{\tau_0}^{\tau_{\mathrm{f}}} \underbrace{\lambda(\tau)\Big(P^{\mathrm{grid}+}(\tau) - k^{\mathrm{pr}} P^{\mathrm{grid}-}(\tau)\Big)}_{\text{power trading profit}} \mathrm{d}\tau \\ &\quad + \mathbb{E}_{\xi}\left[\int_{\tau_0}^{\tau_{\mathrm{f}}} \left(\underbrace{C^{\mathrm{fuel}}\, Q^{\mathrm{in}}(\tau, \xi)}_{\text{fuel cost}} + \underbrace{C^{\mathrm{dev}}(\tau, \xi)}_{\substack{\text{power}\\ \text{deviation cost}}}\right) \mathrm{d}\tau + \underbrace{V(\xi)}_{\text{cost-to-go}}\right]. \end{aligned}$$

The first term is deterministic, since P^{grid} is the unique profile the CHP owner has to determine in this stage. The last three terms instead, depend on stochastic variables, since the system dynamics are stochastic due to the stochastic environment. Therefore, we choose to minimise the expected value of these cost terms.

The power deviation cost term is introduced since a unique power profile might be impossible to deliver under all possible realisations of the stochastic environment. In other words, in extreme situations it might be either physically impossible or economically disadvantageous to deliver the chosen power profile. Therefore, this term accounts for the cost of possible future imbalance in accordance with the imbalance settlement procedure [15]. We assume a one-price settlement procedure, which we formulate by defining variable Δ and the integrand C^{dev} as follows

$$\begin{aligned} &\Delta(\tau, \xi) := P(\tau, \xi) - P^{grid}(\tau) - P^{mg,rl}(\tau), \\ &C^{\mathrm{dev}}(\tau, \xi) \geq b_1 \Delta(\tau, \xi), \qquad C^{\mathrm{dev}}(\tau, \xi) \geq b_2 \Delta(t, \xi). \end{aligned}$$

This formulation is further discussed in Section III-B and illustrated in Figure 5.

The last term is a cost-to-go used to describe the value of the storage content. Without any limitations, the heat and electric energy in the storages would be extracted completely, thus leaving no energy for the next day. We build the cost-to-go function, as is typically done in dynamic programming, and use it as terminal cost. We approximate this function based on the profit possibilities after this period where we assume the same conditions. Obtained function is then approximated with an epigraph formulation.

The optimisation problem can then be formulated as

$$\min_{X(\cdot),U(\cdot)} \quad J_s(X(\cdot), U(\cdot), \Theta(\cdot)) \tag{9a}$$
$$\text{s.t.} \quad X(0) = \hat{X}(0), \tag{9b}$$
$$\dot{X}(\tau) = f(X(\tau), U(\tau), \Theta(\tau)), \tag{9c}$$
$$h(X(\tau), U(\tau), \Theta(\tau)) \leq 0. \tag{9d}$$

In order to solve the infinite-dimensional Problem (9), the control vector is parametrised as piecewise constant over intervals of 1 hour and the dynamics are discretised using a first-order explicit Euler integration. The dynamics being slow, the integration error has been found to be negligible. Because of the stochastic nature of the parameters, the problem is still infinite-dimensional. We solve this issue by sampling the stochastic distribution of the parameters, as outlined in Section IV.

The strongly nonlinear efficiency η and the discrete states of the CHP unit make this problem a non-linear mixed-integer problem (MINLP). This class of problems is particularly hard to solve and we propose a reformulation to a mixed-integer linear problem (MILP). In order to achieve this we approximate the nonlinearity using the MILP formulation presented in [16]. We then rely on fast MILP solvers to solve the optimisation problem. Modelling of the discrete nature of the CHP unit operation in a optimization framework is described in Appendix A while the piece-wise linear approximation of the non-linear efficiency is described in Appendix B.

B. Real-Time Operation

When operating the system in real-time, the basic objective in real-time operation is to minimize fuel usage while satisfying the actual microgrid's demand for the electricity/heat and ensuring the basic quality of supply constraints. In this setting, it can be optimal to deviate from the scheduled power profile and pay the relative fees if the benefit of doing so is larger than the fee.

The cost is then given by $J_{\mathrm{rt}}(x(\cdot), u(\cdot), \theta(\cdot)) :=$

$$\int_{\tau}^{\tau+\tau_{\mathrm{rt}}} \ell\big(x(\tau'), u(\tau'), \theta(\tau')\big)\mathrm{d}\tau' + \underbrace{v_{\mathrm{s}}\big(x_{\tau+\tau_{\mathrm{rt}}}\big)}_{\approx \text{ cost-to-go}},$$

$$\ell\big(x(\tau'), u(\tau'), \theta(\tau')\big) := \underbrace{c_{\mathrm{lin}}^{\mathrm{fuel}}(\tau')}_{\text{fuel}} + \underbrace{c^{\mathrm{dev}}(\tau')}_{\text{power}} + \underbrace{c^{\mathrm{temp}}(\tau')}_{\substack{\text{comfort,}\\ \text{heat storage}}}.$$

The first term in the cost integrand corresponds to the first term in the day-ahead scheduling and is a nonlinear function of p, q^{ext}. Provided that the deviations from the day-ahead scheduling are small, the linear approximation $c_{\mathrm{lin}}^{\mathrm{fuel}} = x^{\top}\nabla_x(\frac{C^{\mathrm{fuel}}}{\eta})|_{x=X}$ is verified to be accurate enough.
The second term in the cost integrand is specific to the real-time operation and accounts for the costs of imbalance, i.e., the cost of deviating from the power profile negotiated in the day-ahead scheduling. We denote the imbalance as

$$\delta(\tau) := p(\tau) - P^{grid}(\tau) - p^{mg,rl}(\tau). \tag{10}$$

According to one-price settlement procedure, if $\delta > 0$, the Microgrid needs to sell additional power at a price which is lower than the day-ahead by the factor b_1. If $\delta < 0$, the Microgrid produces less power than required and needs to buy additional power which is more expensive than the day-head cost by the factor b_2. We formulate this using the epigraph formulation

$$c^{\mathrm{dev}}(\tau) \geq b_1\delta(\tau), \quad c^{\mathrm{dev}}(\tau) \geq b_2\delta(\tau). \tag{11}$$

A depiction of the meaning of the first two terms in the cost is given in Figure 5.

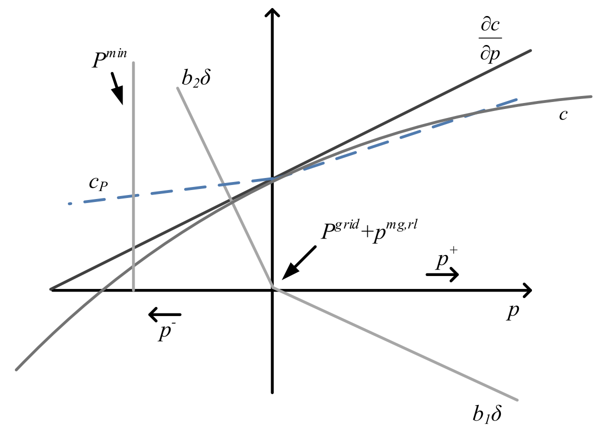

Fig. 5. Economic objective related to the electric power.

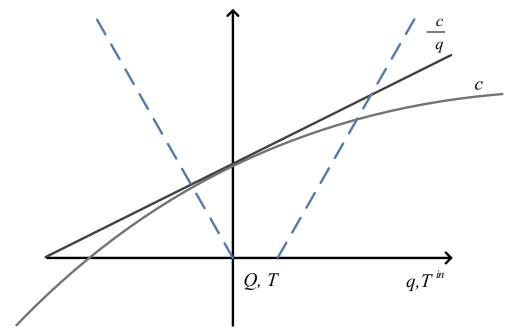

Fig. 6. Economic and tracking objective related to the building temperature and heat extraction.

The third term in the cost integrand is related to the customer satisfaction in terms of guaranteeing the required water temperature is met with a prescribed accuracy. Therefore, deviations from the required temperature which are larger than a prescribed accuracy interval incur some additional cost. The same considerations are also applied to the heat storage temperature. We define $\star \in \{\mathrm{in}, \mathrm{hs}\}$, the prescribed intervals as $[t^{\star,\mathrm{lb}}, t^{\star,\mathrm{ub}}]$. Then, the cost is $c^{\mathrm{temp}} = c^{\mathrm{in}} + c^{\mathrm{hs}}$, with $c^{\star}(t^{\star+} + t^{\star-})$, $c^{\star}$ a prescribed fixed cost, and

$$t^{\star} = T^{\mathrm{in,pred}} + t^{\mathrm{in}=} + t^{\star+} - t^{\star-}, \tag{12a}$$
$$t^{\star+} \geq t^{\star,\mathrm{ub}} \quad t^{\star-} \geq -t^{\star,\mathrm{lb}}, \quad t^{\star=} \in \Big[t^{\star,\mathrm{lb}}, t^{\star,\mathrm{ub}}\Big]. \tag{12b}$$

Note that the cost factor $c^{\star}$ has no direct economic meaning, but can nevertheless be interpreted as the cost incurred by failing to satisfy the customers, which could lead to a contract renegotiation or a loss fo the customer. Figure 6 gives a graphical representation of the economic and tracking objectives related to the real-time building heat supply.

The last term in the cost is built to approximate the cost-to-go of the problem, discussed in Section III-C.

The real-time MPC problem can then be formulated as

$$\min_{x(\cdot),u(\cdot)} \quad J_{\mathrm{rt}}(x(\cdot), u(\cdot), \theta(\cdot)) \tag{13a}$$

$$\text{s.t.} \quad x(\tau) = \hat{x}(\tau), \tag{13b}$$

$$\dot{x}(\tau') = f\big(x(\tau'), u(\tau'), \theta(\tau')\big), \tag{13c}$$

$$h\big(x(\tau'), u(\tau'), \theta(\tau')\big) \leq 0. \tag{13d}$$

The initial condition (13b) is such that, at the beginning of the day $\hat{x}(0) = \hat{X}(0)$. Note that the cost functional implicitly defines additional constraints, as described by, e.g., (12).

In real-time MPC, due to the relatively short prediction horizon, the stochastic parameters can be approximated by a single scenario and the problem becomes deterministic.

C. Value Function Computation

Because the MPC formulation (13) predicts the system evolution in the near future, based on the current realisation of the stochastic parameters, one would ideally want to also introduce information about the future evolution of the system by accounting for its stochasticity. Several approaches are available to deal with this issue. One can, e.g., prolong the prediction horizon beyond τ_{rt} with a stochastic prediction. Because this solution can be computationally too expensive, we propose to solve a stochastic optimal control problem (OCP) at a slower sampling rate and approximate its value function $V_\tau^{\tau_{\mathrm{f}}}(\hat{X}(\tau), \Theta)$ by $V_{\mathrm{s}}(x)$. In order to do so in a consistent way, we need to consider the discretization strategy adopted for the stochastic OCP. First, we remark that the adjoint variable $\mu(\tau)$ associated with the system dynamics is the gradient of the value function evaluated at the optimal trajectory. Second, we consider the sampling strategy used to approximate the stochastic distribution of the parameters. Because we use single trajectories rather than a scenario-tree approach, each sample s of the parameters yields a single state and control trajectory, which we denote by $X_s(\tau)$, $U_s(\tau)$ with associated adjoint $\mu_s(\tau)$ and cost-to-go $C_{\mathrm{s}}(\tau)$. Then, we can approximate $V_\tau^{\tau_{\mathrm{f}}}(\hat{X}(\tau), \Theta)$ by the epigraph formulation obtained by introducing variable v_{s} and constraints

$$v_{\mathrm{s}} \geq \mu_s(\tau + \tau_{\mathrm{rt}})^\top x(\tau + \tau_{\mathrm{rt}}) + C_{\mathrm{s}}(\tau + \tau_{\mathrm{rt}}). \tag{14}$$

If the cost-to-go was convex, (14) would arguably be a good approximation of the exact cost-to-go of the discretized problem. Since we know that the problem is nonconvex, this approximation inevitably introduces some form of conservatism. While a nonconvex continuous piecewise linear approximation can in principle also be computed by using the adjoint variables, it can be hard to compute it, especially for a high dimensional state-space.

Note that, if a scenario-tree approach is used, constraint (14) can be written by taking the expected value of the right hand side at the discrete time instants when branching of the tree occurs.

Because we have discretised the OCP, the solver yields the value of all variables at discrete times $\tau_k = \tau + k\tau_{\mathrm{s}}$, where τ_{s} is the sampling time. In order to recover the cost and adjoint variables at intermediate times, we use the fact that we discretise the system using a single step of forward Euler integration. Therefore, we can compute any state, control or adjoint variable A, as $A(\tau) = (\tau - \tau_k)A(\tau_k) + (\tau_{k+1} - \tau)A(\tau_{k+1})$, for $\tau \in [\tau_k, \tau_{k+1}]$. For a discretisation over N control intervals, the cost at time $\tau \in [\tau_k, \tau_{k+1}]$ is approximated as

$$C_{\mathrm{s}}(\tau) = (\tau - \tau_k)\ell(X_{\mathrm{s}}(\tau), U_{\mathrm{s}}(\tau), \Theta_{\mathrm{s}}(\tau)) + \tau_{\mathrm{s}} \sum_{j=k}^{N} \ell\big(X_{\mathrm{s}}(\tau_j), U_{\mathrm{s}}(\tau_j), \Theta_{\mathrm{s}}(\tau_j)\big).$$

Finally, we remark that, if one wants to avoid solving the stochastic optimal control problem in a receding horizon fashion, the proposed cost-to-go approximation strategy can also be applied to the approximation of the cost-to-go of the day-ahead scheduling problem (9), at the expense of not accounting for the most up-to-date predictions of the stochastic parameters.

IV. Stochastic Parameters Modelling

The sampling of the model parameters is performed using multiplicative autoregressive moving average (ARMA) models of different orders [17]. ARMA models are typically used in modeling/forecasting prices in general as well as for a special case of electricity prices [18]. A similar approach is adopted here with the following modification related to non-Gaussian processes. Namely, we deploy a Copula theory, which provides a general framework, in accordance with the models presented in [19]. The ARMA models and the described transformation provide a framework for the simulation of any stochastic process, preserving its statistical properties.

The parameters considered in this model are:

- Electricity prices λ;
- Air temperature T^{out};
- Solar irradiance I;
- Microgrid residual load $P^{\mathrm{mg,rl}}$;
- Consumer preferences in temperature $T^{\mathrm{in,min}}$, $T^{\mathrm{in,max}}$.

We assume that an estimate of the parameters' probability density function (pdf) is available from historical measurements. Measurements at the current time are used to refine the parameters estimate. Consequently, at the beginning of the prediction horizon, the pdf will have a strong peak around the latest estimate, since the uncertainty is small over short horizons; as the horizon becomes longer, the uncertainty increases. The effect of uncertainty on the pdf over time has been investigated in [20] for wind speed and in [21] for irradiation. In this paper, we have adopted the same approach in order to generate the samples of the pdf to be used for optimal control.

A scenario-based description is not adopted in the case of electricity prices: since the scheduled power P^{grid} is not a scenario-dependent variable, we use expected prices, which equal the forecast price input.

Because the residual load depends on both the electric load and the renewable generation, we generated electric load scenarios based on the procedure outlined above, while we generated PV production scenarios by using the energy transformation of the irradiance scenarios as detailed in [22]. While at the level of urban/rural settlement, it is reasonable to only

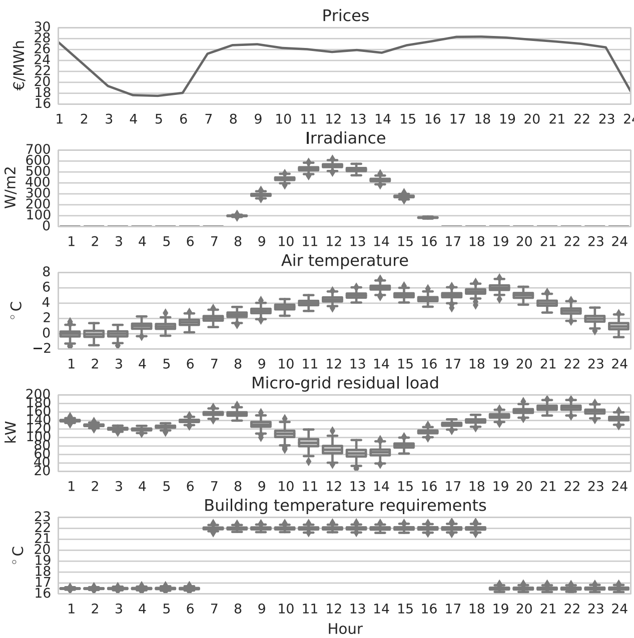

Fig. 7. Stochastic parameters scenarios and electricity price forecast.

consider PV power-plants, the generalisation to a general renewable case is straightforward.

The scenarios in input parameters simulated through the presented procedure are depicted in Fig. 7.

In order to reduce the computational complexity in the scheduling, the number of scenarios should be kept relatively low. In order to decide how to select the scenarios, it is important to keep in mind the two purposes that these data have: (a) to accurately characterize the expected economic benefits, and (b) to capture the extreme state trajectories in order to make robust preparations for the real-time control. For the first purpose, a classical scenario-reduction technique is adopted here based on the technique proposed in [23]. This technique is based on deleting one of the two most similar scenarios in terms of some probability distance measure and assigning its probability to the remaining one. This sequence is repeated until a sufficient reduction is achieved in terms of impact on the objective.

This set of scenarios is augmented with the addition of the two most distant scenarios with a negligible probability assigned. The low probability will diminish their impact on economic valuation while they will still be accounted for in the model, especially for constraint satisfaction. This allows us to account for extreme situations, thus increasing robustness of the scheme.

V. Case Study

For a case study, a simple residential area is considered, consisting of several buildings with different orientation. The input stochastic parameters scenarios are shown in Fig. 7. Optimization is solved over a reduced number of scenarios using GUROBI solver [24] within Python environment.

A. Results

Figure 8 depicts scenario-dependent trajectories of the total heat input and the building heat together with storage and

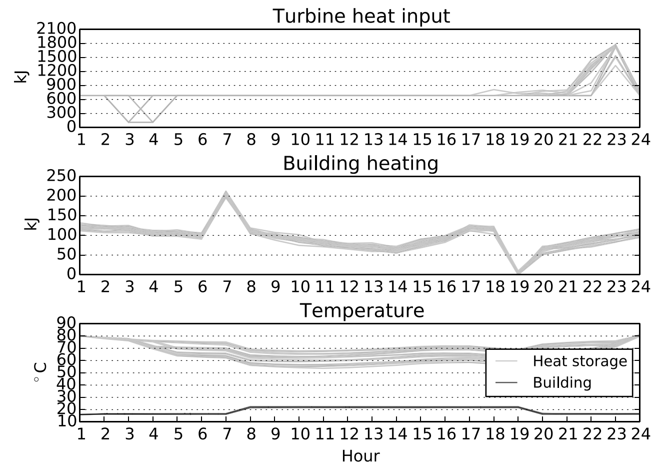

Fig. 8. Heat-related states and controls.

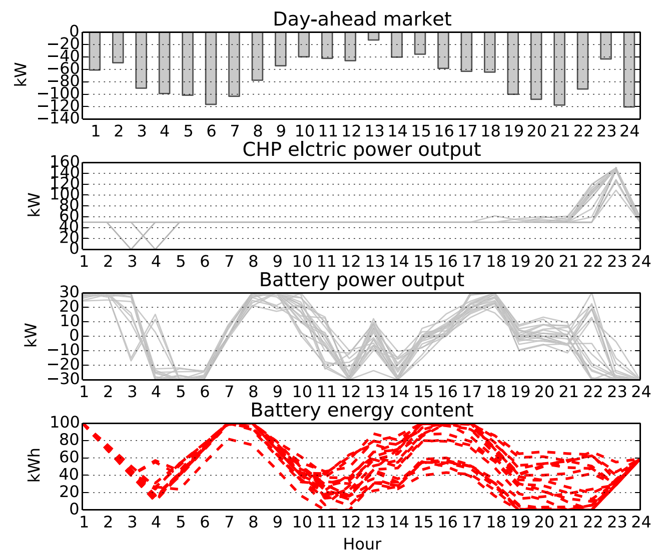

Fig. 9. Electric component of the system.

building temperatures. The last sub-figure depicts the trajectories of the heat-storage temperature and the building temperature. Obviously, the heat input and the heat extraction are changed in each scenario in order to satisfy costumer needs in terms of building temperature. This temperature is kept on its lower bound of the desired range due to the lower costs.

Figure 9 depicts the scheduled trading quantities. It can be seen that the Microgrid prefers buying power from the grid over producing the same with the CHP unit. The avoidance of producing power with CHP unit is a result of lower prices compared to the fuel costs. On the other hand, due to the necessary heat production, the CHP unit produces minimal possible electric power based on the operating range (in several scenario unit is being shut down at certain moments). On the other hand, battery power is fully utilized due to the low costs.

Figure 10 depicts the real-time control performed with the proposed economic MPC scheme. The first two sub-figures show the control of the heat storage temperature and the

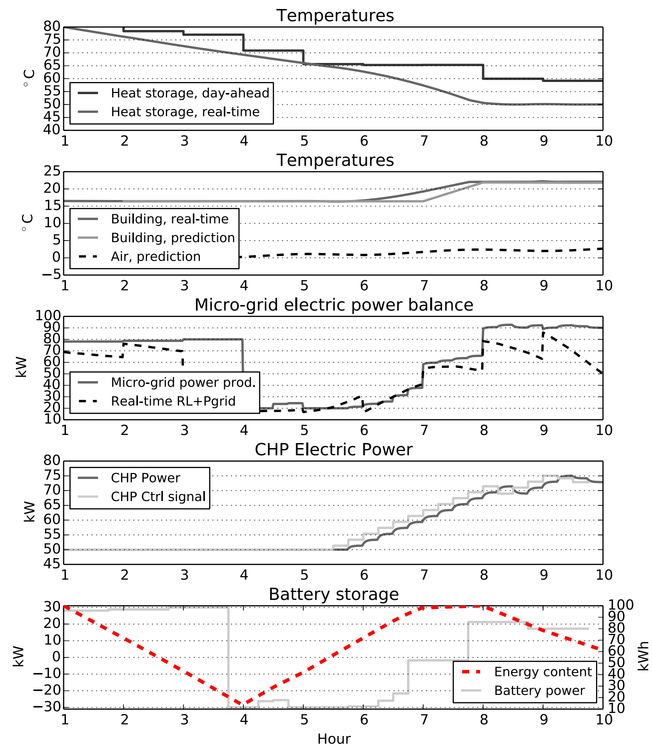

Fig. 10. Real-time economic MPC.

control of the predicted requirements of the building temperature. The third sub-figure depicts the Microgrid power balance between controllable sources and passive elements. Controllable units (CHP and battery) do not follow the residual load $+P^{grid}$ perfectly due to the relation between the fuel costs and the imbalance costs discussed in the previous section.

B. Performance Analysis

In order to validate the proposed control framework we will compare it with a typical tracking control used in these kinds of systems. Classic scheduling+tracking schemes determine the optimal trajectories in the scheduling phase which are followed in the tracking phase without accounting for any economic criteria. The main benefit of the proposed scheduling+economic MPC framework is the ability to economically take into account the latest updates on the parameters forecasts. We have applied the classic scheduling+tracking scheme on the same case study and compared the results. We found that the proposed scheme can save on average 5.5% in costs daily.

It is important to specify that the performance results presented here are related to the specific case study. Obviously, the benefits of the proposed scheme depend on the temporal resolution of the scheduling and real-time control. The benefits are in direct relation with the uncertainty in the scheduling. Therefore, though it is not possible to draw any general conclusion on the performance gains of the proposed scheme, the case study indicates that such gains can be substantial.

The proposed model adopts a centralized control structure which typically results in an increased computational complexity. In order to evaluate the computational efficiency of the proposed model the size of the case study is increased in terms of the number of buildings. Table VI-B depicts the results of the analysis performed with a 2.6GHz i5 CPU. The necessary computational time, even in the case of a large residential area, is significantly lower than the thermal process control resolution.

TABLE I
COMPUTATIONAL EFFICIENCY

Number of buildings	1	10	20
Computational time	1.51 s	5.04 s	12.09 s

VI. CONCLUSION

The model presented in this paper is an improvement of a traditional CHP scheduling model towards a model-based scheduling. Such an approach is more suitable for the Microgrids and other modern concepts with active components. Smart building is an example of an active component which can provide a central management system with the information on conditions and preferences in the building. Based on these feedbacks, the central management can control the heat extraction. For the purpose of subsequent real-time control, economic MPC is adopted and together with model-based scheduling provides a general set-up for power management and control for a CHP-based Microgrid.

The basic capabilities of this set-up are demonstrated on an illustrative case study. The results depict system states trajectories following different input scenarios. The results clearly depict the transition between the scheduling and real-time control and their connection. The set-up captures the phenomena related to two different temporal scales (daily trading and real-time operation) and connects them intuitively. Unlike the traditional scheduling with tracking MPC, this set-up accounts for the economic aspects in each phase.

VII. FUTURE WORK

For future work, the proposed method could be further validated on a more comprehensive set of cases. It would be useful to analyse the effect of the Microgrid size and structure on the performance of the proposed control scheme. In this case study a daily and real-time decision-making/control is considered. However, different temporal resolutions could be a better set-up for the proposed scheme.

Epigraph approximations of the value functions could be studied further together with a non-linear form of the optimisation set-up. In a nonlinear form, instead of epigraph, value function could be approximated with polynomials.

APPENDIX A
OPTIMIZATION FORMULATION OF CHP ON-OFF STATUS

Typical CHP unit can lower its production down to a certain level after which it must be shut down. This can be modelled with a time t and scenario ξ dependent binary variable $z_{t,\xi}$ that can be introduced in the operating range constraints in order to enforce shut-down.

$$a_k P_{t,\xi} + b_k Q^{ext}_{t,\xi} \geq z_{t,\xi} c_k, \quad k = 1, \ldots, 3 \tag{15}$$

Other important operating constraints in a typical CHP unit are minimum times (minimum up time MUT and minimum down time MDT) a CHP plant has to remain in a binary state (on-off) after the state changed. This can be incorporated with the following constraints:

$$1 - z_{t,\xi} \geq \sum_{t'=t+1-MDT}^{t} sd_{t,\xi}$$
$$z_{t,\xi} \geq \sum_{t'=t+1-MUT}^{t} su_{t,\xi} \tag{16}$$

where $sd_{t,\xi}$ and $su_{t,\xi}$ are binary variables describing shut-down and start-up status. In order for logic to hold, the following constraint is added:

$$z_{t-1,\xi} - z_{t,\xi} + su_{t,\xi} - sd_{t,\xi} = 0 \tag{17}$$

Additional information on optimization based modelling approaches can be found in [25].

Appendix B
Piece-Wise Linear Approximation for MILP

Approximating non-linear function with a piece-wise linear formulation is an essential component in MILP based optimization. Depending on the convexity and dimension of the function this procedure can range from a simple approximation without integers to a rather complex one with integers which ensure the correct choice of the segments.

Energy conversion function $\eta(P, Q^{ext})$ adopted in Equation (2) is a nonlinear function of two variables. Such a function requires complicated integer-based approximation presented in [16]. the following set of equations enable this approximation:

$$P = \sum_{i=1}^{m} \sum_{j=1}^{n} P_i \pi_{ij} \tag{18}$$

$$Q^{ext} = \sum_{i=1}^{m} \sum_{j=1}^{n} Q_j^{ext} \pi_{ij} \tag{19}$$

$$Q^{in,\eta} = \sum_{i=1}^{m} \sum_{j=1}^{n} \eta_{ij} \pi_{ij} \tag{20}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} (u_{ij} + \omega_{ij}) = 1 \tag{21}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \pi_{ij} = 1 \tag{22}$$

$$\lambda_{ij} = u_{i,j-1} + u_j^{i-1} + u_{i,j} + u_{j+1}^i + u_{i+1,j} + u_j^i \quad \forall i, j \tag{23}$$

The use of this MILP approximation makes the problem mixed-integer, for which efficient solvers are available. Moreover, as the problem formulation includes integer variables due to the on-off CHP status model, we find it reasonable to approximate the original mixed-integer nonlinear problem as a mixed-integer linear by introducing additional integer variables. We remark that, even in the absence of the on-off CHP model, the adopted MILP formulation yielded faster solution times than the corresponding NLP formulation, with the additional advantage of providing global optima as opposed to local ones. Depending on the level of discretization, this formulation can be faster to solve without a significant loss of accuracy. In the case of non-linear function $\eta(P, Q^{ext})$, average difference between exact and approximated function is in the range of 1-3%.

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